THE PHYSICS OF ROCKETS

by

Howard S. Seifert
Mark M. Mills
Martin Summerfield

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JET PROPULSION LABORATORY
GALCIT
California Institute of Technology
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"Fire Arrow"
H. S. Seifert
Physics of Rocket

The Chinese symbol for "rocket" -

literally "fire arrow"

(reading from top to bottom)

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PART I. PRINCIPLES OF ROCKET PROPULSION

Section A. Introduction

1. General Introduction

Although the Chinese are credited with the use of gunpowder rockets as early as several centuries B.C., and Hero of Alexandria invented a steam jet propulsion device about 100 B.C., most of the serious effort to develop rockets has occurred in the last three decades. Goddard[1] in America made a complete study of

[1] Goddard, R. H., "A method of reaching extreme altitudes", Smithsonian Misc. Collections, 71, No. 2, (1919)

rocket performance in 1914. The German all-out rocket program commenced in 1935 culminating in the V-2, which was first fired in September of 1944. Since 1938, intensive rocket research has been carried out by a number of American agencies, including a basic theoretical contribution by Malina[2] in 1940.

[2] Malina, F. J., "Characteristics of the rocket motor unit based on the theory of perfect gases", Jour. Franklin Inst., 230, 433-454 (1940)

The present paper will concern itself only with that type of jet propulsion device designated as a "pure" rocket, i.e., a thrust producer which does not make use of the surrounding atmosphere. This restriction excludes propulsive duct devices such as the "turbojet" engine used in jet propelled airplanes of the P-80 type. No attempt will be made to discuss the aerodynamics of bodies moving at supersonic speeds, the electronic problems of rocket missile guidance and control, the measurement of physical quantities in the upper atmosphere, or the properties of weapon rockets. Even omitting these interesting fields, the science of rocketry embraces many phases of physics and chemistry, as will appear in later sections.

2. Physical Nature of Reaction Propulsion

A rocket is a rigid container for matter and energy so arranged that a portion of the matter can absorb the energy in kinetic form and subsequently be ejected in a specified direction. The matter, originally stationary relative to the container, is emitted usually as a continuous fluid with an "exhaust" velocity v and a mass flow rate $m = dM/dt$, thus experiencing a rate of change of momentum mv. This rate of momentum change is transmitted to the residual solid portion (total mass M) of the rocket as a reaction force or "thrust"

$$F = mv, \tag{1}$$

where the vectors F and v are in opposite senses and m is negative, since it represents decreasing M. Eq. (1) assumes that the mass is exhausted into a vacuum. Thus the basic mechanics of a rocket appear disarmingly simple. It is a device for producing maximum rate of change of momentum (and hence thrust force) by means of a minimum expenditure of mass. A brief calculation shows that for a constant thrust, the rate at which kinetic energy is supplied to the expended mass varies inversely as the mass flow rate. Since an effective rocket must spend mass frugally, it squanders energy prodigally. This high rate of evolution of energy implies that the ejected matter is subjected to elevated temperatures and consequently is in the gaseous state. In the discussion which follows, the exhaust products will be assumed to follow the laws of ideal gases. The goal of a rocket "motor", then, will be to convert random thermal energy of a gas into an ordered state in which all the molecules have been collimated in a specific direction. In this ideal condition their macroscopic momentum vector will be a maximum and their scalar temperture and pressure essentially zero. A rocket which must operate in the earth's atmosphere cannot achieve this perfect efficiency which would require expansion into a vacuum. The expansion process must stop when the pressure of the emerging jet equals that of the surrounding atmosphere. Consequently the efficiency of the motor has limitations.

3. Criteria for Rating Rocket Performance

The rocket is unusual in the field of propulsion devices in that its thrust is independent of its velocity and does not require the presence of surrounding matter. This contrasts with the airplane power plant, for example, the thrust of which decreases with increasing relative velocity and decreasing density of the atmosphere. Conventional motors normally propel their loads at a constant speed; rocket motors usually are accelerating a free body of rapidly decreasing mass. The goal of a conventional motor is to exert a force through a distance; that of a rocket, to exert a force during a time interval, i.e., to achieve a given terminal velocity. Therefore, the impulse produced (or momentum change) is a more significant parameter in rating rockets than energy dissipated, and the thrust per unit weight rate of flow - called the "specific impulse" - is a more logical measure than the power generated. The propulsive power developed by a rocket is proportional to its speed. For example, the German V-2 rocket at its maximum speed of 5000 ft/sec develops over half a million horsepower, while the power immediately after take-off is relatively low.

The following quantities are commonly used in rating rocket performance:

Total impulse $I = \int F dt$ = thrust x duration (lb-sec)

Impulse is a convenient measure of the performance of a rocket because it is found that the amount of propellant necessary is the same for the same total impulse, no matter whether it is delivered as a large thrust for a short time or a small thrust for a long time.

$$\text{Specific impulse } I_{sp} = \frac{Ft}{M_{pg}} = \frac{\text{impulse}}{\text{propellant wt.}} \quad \text{(sec)} \tag{2}$$

$$I_{sp} = \frac{F}{m_{av}g} = \frac{\text{thrust}}{\text{weight flow rate}}$$

By taking the reciprocal of I_{sp} one has the "specific propellant consumption" $w_{sp} = 1/I_{sp}$. Multiplying I_{sp} by g

gives the "effective exhaust velocity" c which is essentially the velocity v appearing in equation (1). c and v differ in that c is defined by the equation

$$c = gI_{sp} \tag{3}$$

and may have a different value from the true velocity of efflux v due to inefficiencies such as the back pressure of the atmosphere on the jet. Although I_{sp} is primarily a measure of propellant performance, its value is affected by the geometrical design of the rocket, the combustion pressure, and the external atmospheric pressure. This must be remembered when comparing propellants.

$$\text{Impulse-Weight Ratio} = \frac{Ft}{(M_p + M_o)g} = \frac{\text{impulse}}{\text{total weight}} \tag{4}$$

The impulse-weight ratio, dimensionally the same as specific impulse, is a measure of the performance of the rocket including the non-expendable mass M_o. It indicates the excellence of the overall design of container-plus-propellant as a unit. Certain rocket propellants whose specific impulses are higher than others may lose their advantage when compared on the basis of impulse-weight ratio, due to their low density.

Section B. Dynamics of Rocket Jets

1. Basic Thermodynamical Relations

In this and the following sections quantitative relations will be presented for the velocity of the gases issuing from the exhaust nozzle of a rocket, as well as the magnitude of the thrust therefrom. For the most part the theory will be developed from first principles for the sake of clarity and continuity, even though certain portions are well documented elsewhere.

Up to this point only the relation between thrust and mechanical energy has been considered. Assuming a rocket propellant with a certain heat of combustion is to be used, one must now investigate the thermodynamic process of converting this heat into useful thrust-producing mechanical energy.

Suppose dq is the additional heat introduced into a unit mass of gas, dE the change in the internal energy[3] of

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[3] Millikan, Roller and Watson, Mechanics, Molecular Physics, Heat and Sound (Ginn and Co., 1937) See p. 264 for definition of internal energy.

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the gas, dV the change in volume of the gas due to this heating and p the pressure at which the process is carried out. pdV is the work done by the gas in expansion. According to the first law of thermodynamics,

$$dq = dE + pdV \tag{5}$$

Here we assume that each quantity is expressed in mechanical units. In general, the internal energy is a function of the gas temperature T, and if Van der Waals forces exist, of the specific volume V also.

We define the specific heat at constant volume, c_v, as the heat added to a unit mass per unit temperature rise when $dV = 0$. Then equation (5) gives

$$c_v = (dq/dT)_V = (dE/dT)_V = \text{const.} \tag{6}$$

Hence c_v is equal to the rate of change of internal energy at constant volume. By transforming equation (5) into a new form one may arrive at an expression for the heat added to a unit mass per unit temperature rise when $dp = 0$, which is defined as the specific heat at constant pressure, c_p. Thus

$$dq = d(E + pV) - Vdp = dH - Vdp \qquad (7)$$

where the quantity $H = E + pV$ is called the heat content or enthalpy of the fluid. From equation (7)

$$c_p = (dq/dT)_p = (dH/dT)_p = \text{const.} \qquad (8)$$

Hence c_p is equal to the rate of change of heat content at constant pressure.

We shall now relate the "mechanical" velocity v of a gas to its thermodynamic attributes of pressure, temperature, and density. Mechanical velocity is used here in the sense of bulk or gross velocity as distinguished from random thermal motion of the molecules. A form of Bernoulli's equation suitable to compressible fluids will be employed. Consider a stream of gas (Fig. 1) in steady flow, and compute the

Fig 1

acceleration of the gas element located between two closely neighboring cross sections of area A. The fact that the stream is in steady flow does not, of course, mean that individual elements of gas are unaccelerated.

Applying Newton's second law of motion to the gas element of thickness ds, one gets

$$\rho A ds \frac{dv}{dt} = pA - (p + dp)A = -Adp, \qquad (9)$$

where ρ = density and p = pressure. The velocity of gas elements passing by any fixed point on the stream will not change with time in steady state flow. However, velocity will change with position along the stream. Thus

$$\frac{dv}{dt} = \frac{\delta v}{\delta t} + \frac{\delta v}{\delta s} \cdot \frac{ds}{dt} = 0 + v\frac{dv}{ds} \tag{10}$$

combining (10) and (9)

$$-dp = \rho v dv \tag{11}$$

The negative sign indicates that velocity increases as pressure decreases. Since we are considering a unit mass, $\rho = 1/V$, where V is the specific volume. If we use this substitution in equation (11) above, which is essentially Newton's second law, and combine the result with the first law of thermodynamics in the form of equation (7), there results

$$dq = dH + vdv = d(H + \tfrac{1}{2}v^2) \tag{12}$$

Since H is the heat content per <u>unit</u> mass, the term $\frac{1}{2}v^2$ is equivalent to kinetic energy. Therefore, the thermodynamic quantities have been combined with the gross mechanical velocity in a useful and fundamental relation. If we assume further that the stream shown in Fig. 1 is undergoing an essentially adiabatic process - for example, expansion through a rocket nozzle - then dq = 0, and equation (12) can be integrated to

$$H + \tfrac{1}{2}v^2 = \text{const.} \tag{13}$$

Equation (13) states that the <u>sum of the heat content and the kinetic energy of a gas is constant in steady, adiabatic flow.</u> Fortunately, this is the case which is applicable to most rockets.

2. Adiabatic Flow

In order to derive quantitative relations describing adiabatic flow i.e., ($dq = 0$), it now becomes necessary to restrict the properties of the compressible fluid to those of an ideal gas. By definition, the internal energy E and enthalpy H of an ideal gas are functions of temperature only. Thus the first ~~thermo-dynamic~~ thermodynamic law can be written, combining equations (5) and (6) or (7) and (8), in the forms

$$c_v dT + p dV = 0 \tag{13}$$

$$c_p dT - V dp = 0 \tag{14}$$

Eliminating dT from equation (13) and (14), and introducing the ratio of the specific heats $\gamma = c_p / c_v$, there follows

$$\gamma \frac{dV}{V} + \frac{dp}{p} = 0 \tag{15}$$

Up to this point there have been no restrictions on c_p and c_v which may be either variable or constant. If we now assume that their ratio γ is constant, equation (15) is integrable to the well known[4a] adiabatic relation

$$pV^{\gamma} = \text{const.} \tag{16}$$

[4a] See p. 276 of Ref. 3

The definition of an ideal gas is completed by requiring that in addition to E being a function of T only, the gas follow the equation of state for unit mass

$$pV = (R_{univ}/M)T = RT \tag{17}$$

where M[4b] is the effective molecular weight of the gas and R_{univ} is a universal constant with the value 1543 ft lb/mol deg F.

[4b] M must be given in mass (not weight) units in Eq.(23) et seq. The usual M in lb-mols must be divided by g

By taking the difference of equations (13) and (14) and combining this with equation (17) we have

$$(c_p - c_v)dT = Vdp + pdV = d(pV) = RdT$$

or $$c_p - c_v = R \tag{18}$$

Combining this result with the definition $\gamma = (c_p/c_v)$ leads to a pair of relations which will later be useful

$$c_p = \frac{\gamma}{\gamma - 1} R \tag{19a}$$

and $$c_v = \frac{1}{\gamma - 1} R \tag{19b}$$

By manipulation of the equation of state (number(17) and the adiabatic law (number(16) one may express either pressure, temperature or density (equivalent to volume) in terms of one of the remaining two variables. Furthermore, the ratio of, say, temperatures at any points 1 and 2, in an adiabatic cycle may be expressed as a simple power of the ratio of any other variable at the same points. These relations are particularly useful in evaluating a parameter at any point in the rocket nozzle in terms of the value of the same quantity in the combustion chamber, at which point it is more readily measured. For instance

$$T/T_c = (p/p_c)^{\left[(\gamma - 1)/\gamma\right]} \tag{20}$$

where the subscript c refers to the combustion chamber conditions. Since $(\gamma - 1)/\gamma \doteq 1/5$ for a rocket exhaust, equation (20) shows that a relatively large change in pressure gives only a small change in temperature as the gases expand adiabatically through the exhaust nozzle. This equation illustrates also the importance of the parameter γ in the dynamics of compressible fluids.

3. Velocity Obtainable by Adiabatic Expansion

We have now developed the necessary relations for expressing the exhaust velocity of the gases in a rocket nozzle explicitly in terms of temperature or pressure. From the definition of c_p in equation (8) and the fact that cp is constant in a perfect gas we have by integration

$$H = c_p T + H_o \tag{21}$$

where H_o is a constant of integration.

Referring to Fig. 2, let us denote the pressure and

Fig. 2

temperature in the combustion chamber of a rocket by p_c and T_c, and those at the exit of the nozzle by p and T. Since the velocity of the gas in the combustion chamber is small, we have from equation (13)(the Bernoulli or energy equation) and equation (21) above

$$v^2/2 - 0 = c_p(T_c - T) \tag{22}$$

But c_p may be expressed in terms of general constants by equation (19a) giving

$$v^2 = [2\gamma/(\gamma-1)]RT_c\,(1 - T/T_c) \tag{23}$$

Using the equation of state (17) and the adiabatic relation Eq. (20) we can replace temperatures by pressures and densities, which are easier to determine, and finally arrive at an expression for the exhaust velocity

$$v = \left\{ 2\left[\gamma/(\gamma-1)\right](p_c/\rho c)(1-\left[p/p_c\right]^{(\gamma-1)/\gamma}) \right\}^{1/2} \qquad (24)$$

Here ρ_c is the density of the gas in the combustion chamber. A number of interesting conclusions may be drawn from equation (24). The factor involving p/p_c increases toward unity as p/p_c approaches zero. Thus, by expanding the gas to lower pressure, the exhaust velocity may be increased. The maximum v_{max}, is obtained if $p = 0$, i.e., by expanding into a vacuum. In this case, all the heat energy of the gas is converted into kinetic energy. From equation (24) the value of v_{max} is

$$v_{MAX.} = (2\left[\gamma/(\gamma-1)\right]\left[p_c/\rho_c\right])^{1/2} \qquad (25)$$

From equation (24) one can specify some of the desirable properties of the exhaust gas by which it is possible to obtain high discharge velocity v and hence efficient production of thrust. For a given rocket application, there is a definite design ratio p/p_c of the pressures in the atmosphere and in the chamber. The atmospheric pressure p may vary from 14.7 psi to zero depending on the rocket flight path. A typical value of p/p_c for liquid propellants is 14.7/300. For p/p_c fixed, equation (24) shows that v increases as γ decreases. Therefore it is advantageous to have a gas for which γ is small, although in practice little control can be exercised over this parameter.

To illustrate another desirable property of the gas, one notes in equation (24) that according to the equation of state (number 17) the factor p_c/ρ_c equals $(R_{univ.}/M)T_c$

This indicates that the chamber temperature T_c should be high and the molecular weight M of the products of combustion should be low to secure high values of exhaust velocity v. That M should be low may be seen in another way by using the principle of equipartition of energy[5]. Consider two combustion chambers

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[5]See p. 210 of Ref. 3

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at the same pressure p_c and temperature T_c containing two species of ideal gas molecule, one of mass M, the other of mass 100 M. If the lighter molecule possesses an average kinetic energy of $\frac{1}{2}Mv^2$ then the heavier, according to the equipartition theory, will possess an equal energy of magnitude $\frac{1}{2}(100M)(v/10)^2$. When discharged into a vacuum, the momentum of the lighter will be Mv, that of the heavier (100M)(v/10) or 10Mv. However, for equal mass flow rates 100 lighter molecules will be discharged for each heavy one, so that the net momentum change and hence thrust from the chamber containing the lighter species will be ten times that of the chamber containing heavy molecules, even though they are at the same temperature and pressure and have the same discharge rates. This fact is important in determining the choice of rocket propellants, and favors those which contain a high percentage of hydrogen.

If the two rockets in the above example were of equal thrust rather than mass flow rate, then the mass flow rate of the lighter molecules would be one-tenth that of the heavier, but the power supplied to the lighter molecules would be ten times that to the heavier. This is consistent with the statement made in paragraph 2 of the introduction.

Section C. Flow Through Nozzles

1. Discharge Velocity and the Velocity of Sound

The factor $p_c/\rho_c = RT_c$ in the velocity equation (24) is really a measure of the total heat energy stored in the gas, and has the dimensions of energy per unit mass. For ideal gases this is simply the kinetic energy corresponding to molecules moving with a certain mean velocity. One notes that a wave-like disturbance is propagated through a gas with a velocity which depends upon this same mean molecular speed of thermal agitation. It might be intuitively expected, therefore, that the velocity of sound waves in a gas and the velocity achieved by the molecules of the same gas in free adiabatic expansion would be simply related. A good measure of the random velocity of molecular motion is the so-called velocity of sound[6] defined by

[6] See Ref. 3, page 363

$$a^2 = dp/d\rho \qquad (26)$$

For adiabatic processes, equation (15) and the equation of state reduce equation (26) to

$$a^2 = \gamma \frac{p}{\rho} = \gamma RT \qquad (27)$$

Hence, if we denote the velocity of sound in the combustion chamber by a_c, then equation (25) for the maximum exhaust velocity can be expressed

$$v_{MAX.} = \left[2/(\gamma - 1)\right]^{1/2} a_c \tag{28a}$$

and equation (24) for the exhaust velocity at any pressure p can be expressed

$$v = a_c\left[(2/(\gamma - 1))(1 - \left[p/p_c\right]^{\frac{(\gamma-1)}{\gamma}})\right]^{1/2} \tag{28b}$$

For many rocket propellants $\gamma = 1.25$, and $v_{max.} = 2.828\ a_c$. For a typical case of expansion from 500 psi to 15 psi, the local velocity of sound at the nozzle exit is half of a_c, and the ratio of exit exhaust velocity to the local velocity of sound v/a - called the "Mach number" M - has a value of 5 or 6. Such flow is designated as <u>supersonic</u> flow, and can be achieved only with a properly shaped exhaust nozzle.

2. <u>The de Laval Exhaust Nozzle</u>

It is necessary in designing rockets to express the thrust (or mv) in terms of known properties of the combustion products and pressures. This has already been done for the exhaust velocity in equation (24). It remains to be shown how the mass flow can be expressed in the same terms, and what effect the back pressure of the atmosphere has on thrust, before thrust can be calculated explicitly in terms of dimensions and pressures.

The geometry of the orifice through which the compressible gases escape is important in determining both mass flow and thrust. It is not evident a priori what the shape of this "nozzle" should be. For example an incompressible ~~incomprehensible~~ fluid such as water flowing through a converging-diverging venturi tube first increases in velocity and then decreases, with a maximum at the smallest cross-section. The mass flow rate is proportional to the overall pressure difference. On the other hand a compressible fluid undergoing adiabatic expansion through a similar venturi will behave in the same manner only so long as the velocity at all points is less than the local velocity of sound. As soon as sonic velocity is reached, which occurs first at the narrowest cross-section or "throat", the behavior changes entirely. The mass flow rate (but not the velocity) is now unaffected by any changes in pressure downstream from the throat. This effect is sometimes called "nozzling". Moreover the gas velocity downstream of the throat will <u>increase</u> (become supersonic) to a value determined by the pressure at the nozzle exit. The pressure difference necessary to cause sonic flow is called the <u>critical pressure</u>. All rocket chambers operate well above it. Quantitative relations to verify these statements will now be derived.

Since the mass flow m through every cross section of a nozzle is constant, we have the continuity condition

$$m = f \rho v = \text{const.} \tag{29}$$

where f is the area of the section at the point where density is ρ and velocity is v. It is possible to express both ρ and v in terms of p/p_c, resulting in a relation between the pressure p at any point and the cross-sectional area f at the same point, for a specified mass flow m. The value of ρ may be written, similarly to equation (20), in terms of pressure as

$$\rho = \rho_c (p/p_c)^{\frac{1}{\gamma}} \tag{30}$$

The value of v is given already in equation (24). Using these in (29) above and solving for f gives

$$f = \frac{m}{\rho_c} \left(\frac{p}{p_c}\right)^{-\frac{1}{\gamma}} \left[\frac{2\gamma}{(\gamma-1)} \left(\frac{p_c}{\rho_c}\right)\left(1-\left[p/p_c\right]^{\frac{\gamma-1}{\gamma}}\right)\right]^{-1/2} \tag{31}$$

If the area f is calculated from (31) for a series of steadily decreasing values of p/p_c, it is found that f has a minimum value[7]. This indicates that in order for p to

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[7]Roberts, <u>Heat and Thermodynamics</u> (Blackie & Son, ed. 3, 1940) p. 293

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decrease (and hence v to <u>increase</u>) steadily, the nozzle should be shaped as in Fig. 3.

Fig. 3.

A nozzle of the contour shown in Fig. 3 is called a de Laval nozzle, after the Swedish engineer Carl G. F. de Laval, who first used it to obtain supersonic gas velocities. The fact that a supersonic nozzle must have the convergent-divergent contour may be shown in another way by writing the equation of continuity (29) in differential form as:

$$\frac{df}{f} + \frac{d\rho}{\rho} + \frac{dv}{v} = 0 \tag{32}$$

The differential Bernoulli equation (11) may be rewritten with the help of the definition of the velocity of sound equation (26) as follows:

$$\frac{dp}{\rho} = \frac{dp}{d\rho} \cdot \frac{d\rho}{\rho} = a^2 \frac{d\rho}{\rho} = - v dv \tag{33}$$

By substituting for $\frac{d\rho}{\rho}$ from Eq. (33) into the above continuity equation (32) and remembering the definition of Mach number $M = v/a$, one gets

$$\frac{df}{f} = - \frac{dv}{v} (1 - M^2) \tag{34}$$

Equation (34) shows that if the velocity increases continuously along the nozzle (i.e. dv always positive) then

when v is subsonic	$M < 1$	$df/f < 0$	f is decreasing.
when v is sonic	$M = 1$	$df/f = 0$	(throat)
when v is supersonic	$M > 1$	$df/f > 0$	f is increasing

The point of minimum cross-sectional area f_t is called the <u>throat.</u> The value of the pressure at the throat p_t may be found by differentiating f in equation (31) with respect to p/p_c. In this way one obtains

$$p_t / p_c = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \qquad (35)$$

The temperature at the throat follows from adiabatic relation (20)

$$T_t / T_c = \left(\frac{2}{\gamma+1}\right) \qquad (36)$$

The pressure ratio given by equation (35) is called the <u>critical</u> pressure ratio. Below this ratio the nozzle is convergent throughout, above it the nozzle is convergent-divergent. It can be shown by substituting the critical pressure ratio of equation (35) in the velocity equation (24) that the velocity at the throat v_t is:

$$v_t = \sqrt{\frac{2\gamma}{\gamma+1} \cdot \frac{p_c}{\rho_c}} = \sqrt{\frac{2\gamma R T_c}{\gamma+1}} = \sqrt{2RT_t} = a_t \qquad (37)$$

which is just the <u>velocity of sound at the conditions prevail-in the throat,</u> as may be demonstrated by using equations (27) and (20) with equation (35).

The area of the throat f_t necessary for a given mass flow rate m with a given pressure ratio can be calculated by substituting the value $\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$ for p/p_c in equation (31), providing the pressure ratio is greater than the critical. Thus, the value of pressure p downstream of the throat has no effect on the mass flow rate. This seemingly anomalous phenomenon is not predicted by the application of Bernoulli's equation, but occurs in contradiction to it[8]. It is a consequence

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[8] O'Brien and Hickox, Applied Fluid Mechanics, (McGraw-Hill, 1937) p. 43

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of the finite velocity of sound and the fact that a reduction in pressure below the throat cannot be propagated upstream through the fluid flowing with supersonic velocity. This explanation was first pointed out by Osborne Reynolds[9].

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[9] Osborne Reynolds, Phil. Mag. Vol. 21, 5th Series, p. 185 (1886)

It is interesting to note that although area equations (31) and (34) indicate that a minimum in area f is necessary if the gas velocity v is to increase continuously, and although equation (31) relates pressure p and area f uniquely, it gives no other information as to the actual geometrical shape of the nozzle. This shape is in fact not unique, and other less fundamental considerations such as weight and heat transfer determine the precise contour. Two characteristic nozzle contours are shown in Fig. 4 for (a) low combustion pressure liquid propellant and (b) high combustion pressure solid propellant.

Fig. 4

A brief qualtitative resume of the behaviour of the de Laval nozzle at various pressure ratios will now be given. If one maintains a fixed chamber pressure but gradually reduces the exit pressure to a value only a little below this chamber pressure, for a given nozzle the flow is first all subsonic and the nozzle is actually a venturi tube. The velocity first increases, but decreases again after the throat section. The velocity at the throat section will increase with continued reduction of the exit pressure, and hence the mass flow through the nozzle will also increase. After sonic velocity is reached at the throat section, further reduction in exit pressure will not increase the mass flow. The mass flow will remain constant because the sonic throat velocity remains constant. However, the velocity of exit will be increased by decreasing the pressure at the nozzle exit, with accompanying complicated shock wave[10]

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[10] Durand, Aerodynamic Theory, (Julius Springer, Berlin, 1934-36) Vol. III, pp 213-222.

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patterns, until the pressure just outside the nozzle corresponds to the expansion ratio [11] of the nozzle. Further reduction in

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[11] This refers to the pressure that would normally be reached by a continuous adiabatic expansion to the given exit area of the nozzle, i.e., the pressure computed from equation (31), when f is the exit area. The "expansion ratio" ϵ will be discussed in a later section.

"atmospheric pressure" will not increase the exit velocity, as the additional expansion of the gas takes place outside the nozzle. This last situation may result in an increase in the thrust F due to the pressure effect, as will be shown later. Fig. 5 illustrates how the parameters p, ρ , v, and T vary throughout the nozzle relative to their values in the throat section.

Fig. 5

3. Mass Flow Through The Nozzle

The mass flow through the nozzle can be expressed in terms of chamber conditions and the area of the throat by combining the continuity equation and previously derived relations in the following manner:

From the continuity equation (29)

$$M = f_t \; \rho_t \; v_t \tag{38}$$

where the subscript t refers to throat conditions.

The sonic velocity at the throat v_t has been found in equation (37) and the throat density ρ_t may be expressed in terms of p_t and T_t by the equation of state (17). But p_t and T_t are known in terms of chamber conditions from equations (35) and (36). Making all these substitutions in (38) results in:

$$M = \Gamma' \frac{f_t \, p_c}{\sqrt{\gamma R T_c}} = \frac{\Gamma' f_t \, p_c}{a_c} \tag{39}$$

where $\Gamma'(\gamma)$ is a constant defined by:

$$\Gamma' = \gamma \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \tag{40}$$

It is apparent from (39) that m is independent of quantities downstream from the throat. a_c is not directly measurable, but it will appear later that a_c cancels out in practical design calculations of m. The mass flow m will now be used in calculating the thrust F of a rocket.

Section D. The Magnitude of the Reaction Thrust

1. Calculation of the Net Rocket Thrust

If a rocket were to operate *in vacuo*, the thrust would be simply calculated from the equation $F = mv$, with corrections for the fact that the exhaust stream might be slightly divergent rather than parallel. However, since it is usually immersed in the atmosphere and subject to pressure forces, a more detailed analysis is necessary.

Referring to Fig. 6, we see that the net thrust force F,

Fig. 6

counted positive when acting in a direction opposite the gas velocity, is the vector sum of all pressure forces over the inside and outside surfaces of the solid shell, or

$$F = \int p\,dS = \int_{S_i} p_i\,dS_i + \int_{S_o} p_o\,dS_o \tag{41}$$

where p = absolute magnitude of pressure on motor wall

dS = vector element of wall surface area

Subscripts i and o refer to the inside and outside surfaces respectively. From symmetry the vector integral will be directed along the axis of the motor.

The integral over the outside surface S_o may be evaluated as follows: The resultant force due to the uniform atmospheric pressure p_o on a completely closed vessel at rest is zero. If this force is resolved into that which would act over the plane area fe of the nozzle exit section together with all other external pressure forces on the rocket we have(Fig. 6a).

Fig. 6a and 6b

$$p_o f_e + \int_{S_o} p_o \, d S_o = 0 \qquad (42)$$

The effect of the open area f_e is to create an unbalanced force opposed to the thrust of magnitude - $p_o f_e$ which is thus the value of the integral in equation (42).

The term $\int_{S_i} p_i \, dS_i$ in equation (41) may be evaluated by considering the mass of gas contained in the rocket motor,(Fig. 6b). The momentum theorem requires that the

~~Fig. 6b~~

integral of all the pressures acting over a surface enclosing this mass be equal to the rate of flow of momentum through this surface enclosing this mass be equal to the rate of flow of momentum through this surface, or mv_{ex} where v_{ex} is the average velocity along the symmetry axis at the exit section.

The pressure acting on the gas is $-p_i$ (the reaction to the pressure p_i on the motor wall) and the average pressure opposing the flow through the exit section f_e is p_e (see Fig. 6b; p_e, the pressure at the nozzle exit is not necessarily equal to p_o). Equating the total force on the gas to its time rate of momentum gives:

$$-\int_{S_i} p_i \, dS_i + p_e f_e = -m v_{ex} \tag{43}$$

The negative sign is used before the last term because it is customary to substitute the absolute positive numerical magnitude for m, which is an inherently negative quantity, being the rate of <u>decrease</u> of total mass.

The average velocity v_{ex} in the axial direction is smaller than the true velocity of efflux v_e, which is radial. A factor λ is applied to correct for this divergence of flow such that $v_{ex} = \lambda v_e$. The value of λ depends on the angle of divergence of the nozzle, and can be calculated[12]. For a nozzle of 15° half-angle, $\lambda = .985$.

[12]See Ref. 2, p. 449

The thrust equation (41) may now be evaluated by means of equations (42) and (43)

$$F = \lambda m v_e + (p_e - p_o) f_e \tag{44}$$

We see from equation (44) that the thrust F consists of two terms, sometimes referred to as <u>velocity thrust</u> and <u>pressure thrust.</u> If $p_e = p_o$ all the thrust is velocity thrust, and

It can be shown by evaluating v_e by equation (24) and differentiating F with respect to p_e that F is also a maximum under these conditions. A nozzle for which $p_e = p_o$ is said to be "perfectly" expanded and the value of the corresponding expansion ratio $\epsilon = f_e/f_t$ may be calculated to be:

$$\epsilon = f_e/f_t = \Gamma' \left\{ \gamma (p_0/p_c)^{1/\gamma} \left[2/(\gamma-1)\right]^{1/2} \left[1-(p_0/p_c)^{\frac{\gamma-1}{\gamma}}\right]^{1/2} \right\}^{-1} \quad (45)$$

Equation (45) is derived by substituting equation (39) for mass flow m in equation (31) for area f and simplifying with the equation of state (17) written in the form $p/\rho = RT$. Equation (45) is useful in practical design calculations.

If $p_e < p_o$ the gases are "over-expanded" and the pressure thrust will be negative, although partially compensated by an increased velocity thrust. If $p_e > p_o$ the gases are "under-expanded". Although the pressure thrust will then be positive (in the same sense as the velocity thrust) it will not compensate completely for loss in v_e due to inadequate expansion. Due to the fact that pressure and velocity thrust tend to compensate, the net thrust F is rather insensitive to variations in expansion ratio ϵ. For example a nozzle correctly expanded at sea level gives about 6 per cent less thrust at 40,000 feet than one which is correctly designed for that altitude.

2. <u>Performance Parameters Useful in Design</u>

a. <u>Effective Exhaust Velocity:</u> It is difficult to determine v_e and p_e of equation (44) experimentally. Furthermore, because the expansion is not strictly adiabatic, frictionless and "perfect", an <u>effective</u> exhaust velocity c is defined as

$$c = \lambda v_e + \frac{(p_e - p_o)f_e}{m} \qquad (46)$$

so that (44) may be written in the form

$$F = mc \qquad (47)$$

The quantity c was discussed from another point of view in equation (3). It is the parameter used in all practical design work, although it may differ from the true velocity v_e. It is determined in practice by measuring thrust F and mass flow rate m and using equation (47). The F measured in equation (47) includes pressure and divergence effects, and may differ from the F due to reaction alone, as defined in equation (1).

b. <u>Thrust Coefficient:</u> It is found useful in practice to define the thrust of a rocket in the form

$$F = C_F \, p_c \, f_t \, . \qquad (48)$$

Since all the quantities involved in equation (48) are readily measurable, experimental values of C_F may be found which can then be applied to determine the throat area of a rocket of any desired thrust and chamber pressure. A theoretical expression ($C_{F_{th}}$) will now be found to compare with this experimental value.

If the thrust equation (44) is expressed in terms of pressure by means of mass flow equation (39) and velocity equation (24) one gets:

$$F = \Gamma' \sqrt{\frac{2}{(\gamma-1)} \left(1 - \left[\frac{p_e}{p_c}\right]^{\frac{\gamma-1}{\gamma}}\right)}\; p_c f_t + (p_e - p_o) f_e \tag{49}$$

By dividing this equation by $p_c f_t$ one arrives at a theoretical thrust coefficient

$$C_{F_{TH}} = \frac{F_{TH}}{p_c f_t} = \Gamma' \sqrt{\frac{2}{(\gamma-1)} \left(1 - \left[\frac{p_e}{p_c}\right]^{\frac{\gamma-1}{\gamma}}\right)} + \left(\frac{p_e - p_o}{p_c}\right) \frac{f_e}{f_t} \tag{50}$$

This coefficient $C_{F_{th}}$ has a maximum value for the case of a "perfectly" expanded nozzle in which $p_e = p_o$.

A chart of the value of optimum $C_{F_{th}}$ and corresponding optimum area ratio ϵ for the typical case of $\gamma = 1.25$ is given in Fig. 7. These are the values which obtain when

Fig. 7

$p_e = p_o$. In some applications, such as long-range missiles, the pressure ratio p_c/p_o varies considerably. It is not possible for the area ratio of a fixed nozzle to be correct for more than one pressure ratio, since a variable ϵ or "rubber" nozzle cannot be constructed without undesirable complications. A compromise value of ϵ is often chosen such that ϵ is correct at the altitude reached when half the propellant is consumed.

In a real nozzle the measured C_F of equation (48) is lower than $C_{F_{th}}$ of equation (50). For a perfect nozzle we should have a $C_F = \lambda\ C_{F_{th}}$, however experimental values of C_F are 2 to 4 per cent lower than this due to friction and other effects.

c. <u>Characteristic Velocity</u>: In addition to the thrust coefficient C_F it is useful to have a practical empirical parameter by which the mass flow m may be calculated, since equation (48) gives no information about m. We may rewrite the mass flow equation (39) in such a way as to define a new parameter called the <u>characteristic velocity</u> c*

$$m = \frac{\Gamma'}{a_c} p_c f_t = \frac{p_c f_t}{c^*} \qquad (51)$$

where

$$c^* = \frac{p_c f_t}{m} = \frac{a_c}{\Gamma'} = \frac{\sqrt{\gamma R_{univ} T_c / M}}{\Gamma'} \qquad (52)$$

The quantity c* as defined in equation (52) has the convenient property that we may express the effective exhaust velocity c in terms of it and the thrust coefficient C_F.

$$c = F/m = \frac{F/p_c f_t}{m/p_c f_t} = C_F c^* \qquad (53)$$

Thus, when c* has been measured experimentally for a given propellant burning at a definite pressure, both mass flow m and effective exhaust velocity c may be calculated for that propellant. It may be seen from equation (52) that c* is determined only by properties of the propellant and the throat diameter. Thus, it is independent of exit conditions and may be considered as the parameter indicating the efficacy of the gas generation or combustion process. c* is commonly used as a measure of the merit of the propellant, although its value is affected also by combustion chamber design. By using physico-chemical methods to calculate T_c, γ, and M in equation (52) a theoretical value of c^* may be found which is about 10 per cent higher than the experimental value calculated from measurements of $p_c f_t/m$.

d. Summary of Motor Performance Parameters: Besides the performance parameters c, C_F and c^*, there are two more parameters first introduced in Section A which can now be expressed in terms of the newer quantities. The propellant consumption of a rocket motor is expressed by the specific propellant comsumption w_{sp} in pounds of propellant consumed per second per pound of thrust, or

$$w_{sp} = mg/F = g/c = g/c^*C_F \tag{54}$$

The reciprocal value of the specific propellant consumption is known as the specific impulse or performance index I_{sp} in pounds thrust per pound propellant consumed per second or

$$I_{sp} = F/mg = c/g = c^*C_F/g \tag{55}$$

A typical value and range of values of all these parameters are listed in Table I. An examination of this table will give a certain perspective of rocket performance.

TABLE I.

Summary of Performance Parameters

Parameters	Symbol	Definition	Units	Typical Value	Range of Values
Specific Propellant Consumption	w_{sp}	w/F	sec^{-1}	0.0051	0.0100-0.0036
Effective Exhaust Velocity	c	Fg/w	ft/sec	6300	3300-9000
Specific Impulse	I_{sp}	$Ft/W = c/g$	sec	196	100-380
Nozzle Coefficient	C_F	$F/p_c f_t$	---	1.36	1.1-1.8
Characteristic Velocity	c^*	$p_c f_t g/w$	ft/sec	4630	3000-5000

w = rate of propellant consumption lb/sec
F = thrust, lb
g = 32.2 ft/sec^2

W = total propellant weight, lb
t = total duration of thrust, sec.
p_c = absolute chamber pressure, psia
f_t = nozzle throat area, in^2.

PART I. FIGURE LEGENDS

Fig. 1. Adiabatic expansion of a compressible fluid.

Fig. 2. Thermodynamic parameters in a rocket chamber and in an arbitrary section of the nozzle.

Fig. 3. The converging-diverging supersonic nozzle, showing throat of area f_t and exit section of area f_e.

Fig. 4. Two typical nozzle contours. The upper one is designed to produce 300 lbs thrust by expanding gases from 300 lb/in^2 absolute (liquid propellant) and the lower one the same thrust by expansion from 2000 lbs/in^2 absolute (solid propellant).

Fig. 5. Dimensionless plot against relative area of supersonic nozzle velocity and thermodynamic parameters referred to values at the throat. Gas flows from left to right.

Fig. 6. Pressure forces acting on a rocket motor to produce the thrust. Part (a) indicates the external forces due to atmospheric pressure, and Part (b) indicates the internal forces due to the combustion gases.

Fig. 7. Plot of theoretical nozzle thrust coefficient C_{Fth} and expansion ratio ϵ against pressure ratio for an ideal gas with $\gamma = 1.25$. C_{Fth} approaches 2.1 asymptotocially as ϵ increases.

Fig. 1

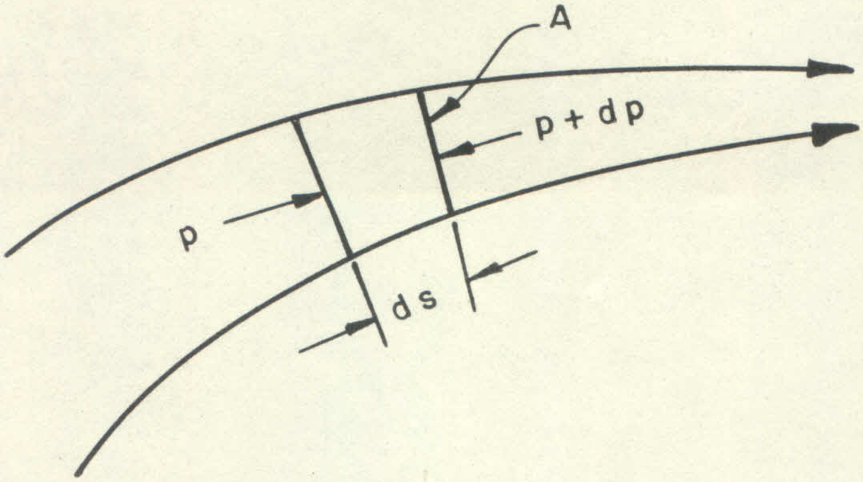


Fig. 1
H. S. Seifert
Physics of Rockets

Fig. 2

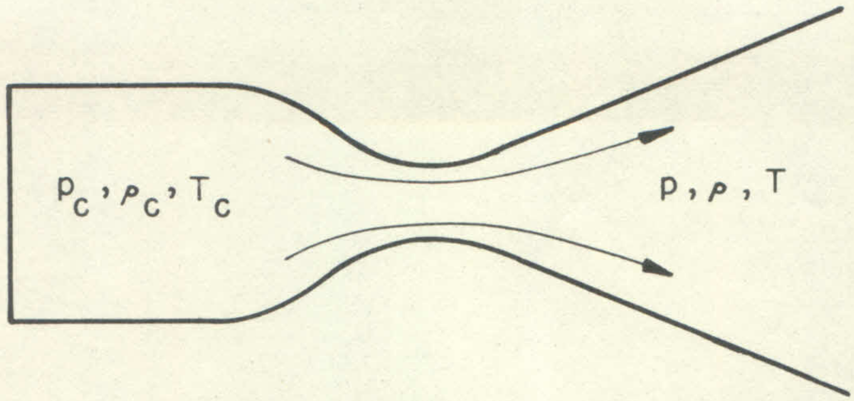
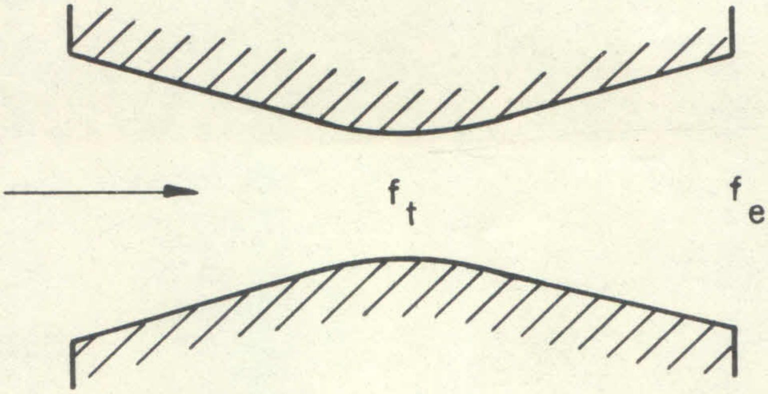


Fig. 2
H. S. Seifert
Physics of Rockets

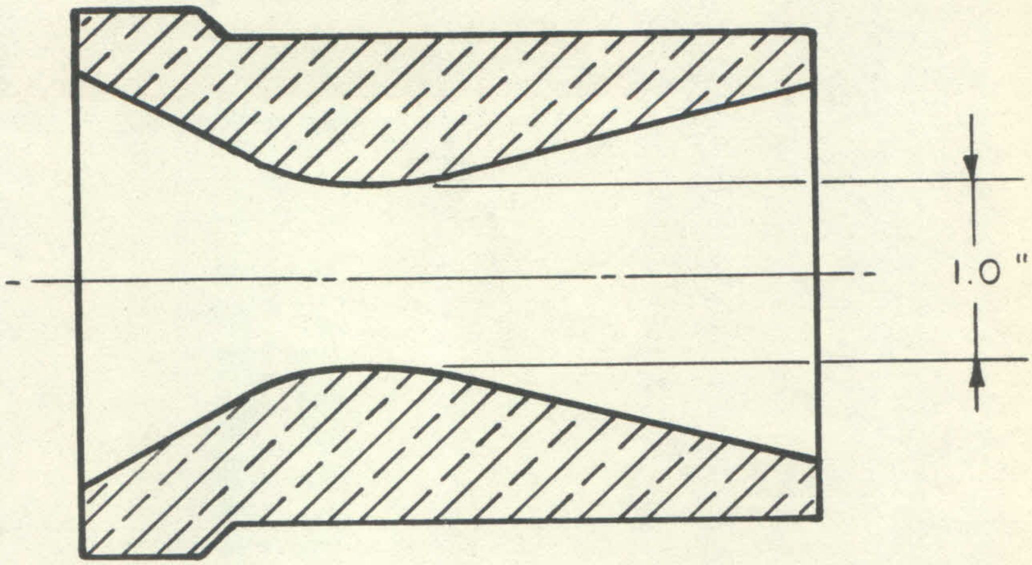
Fig. 3



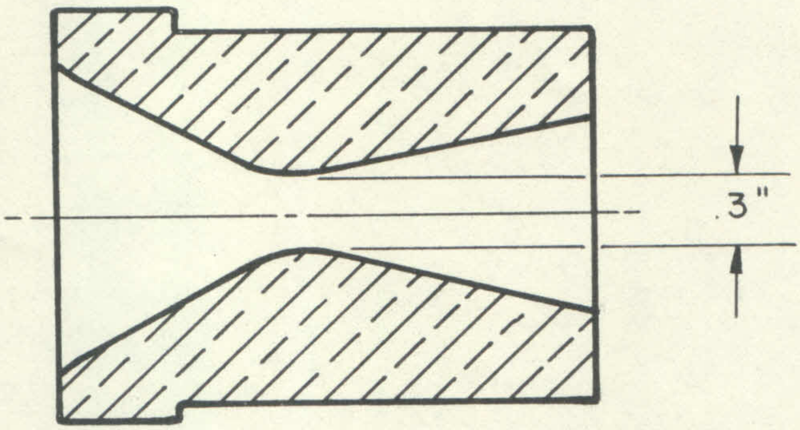
THE DE LAVAL NOZZLE

Fig. 3
H. S. Seifert

Fig. 7



LOW PRESSURE RATIO



HIGH PRESSURE RATIO

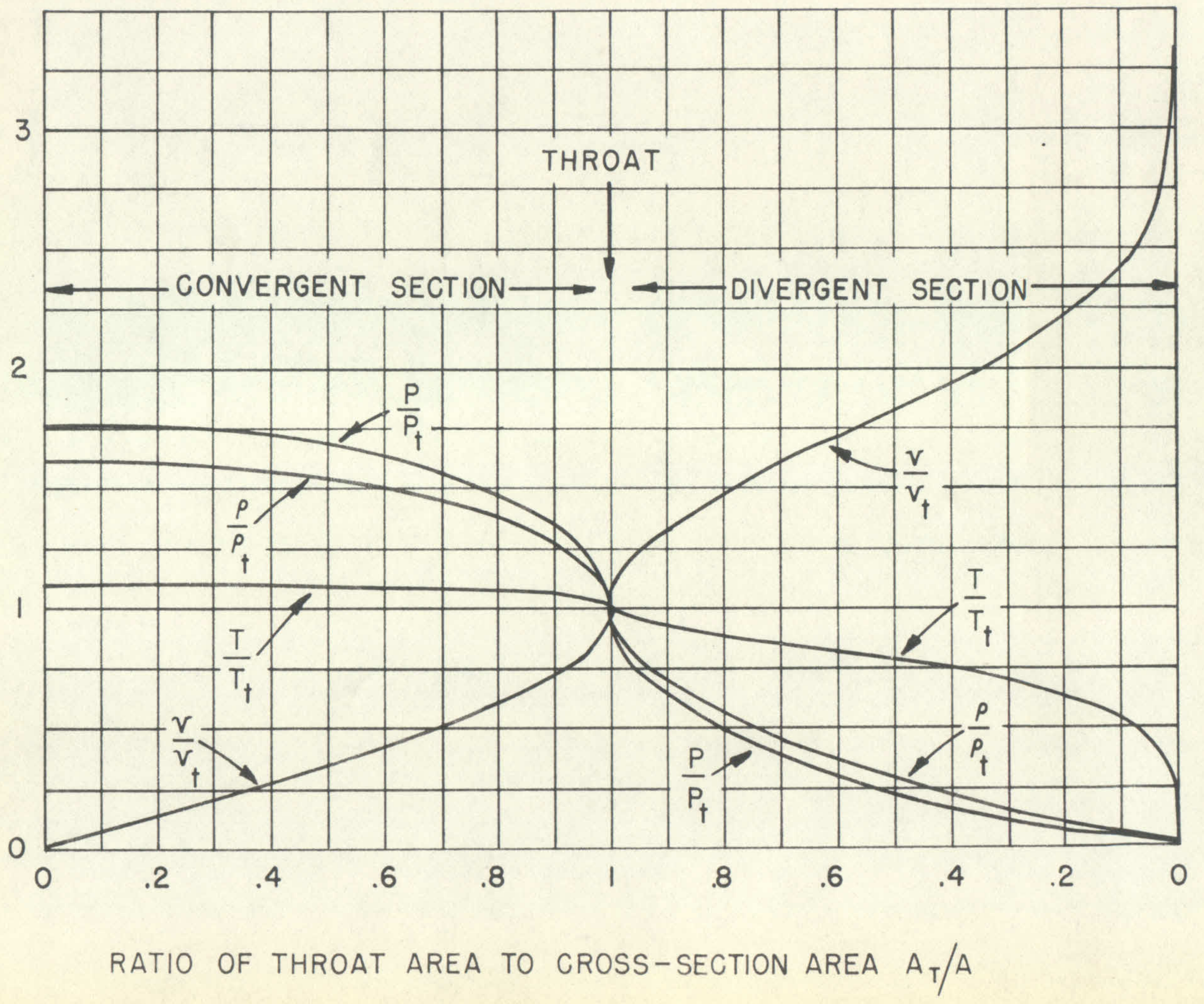
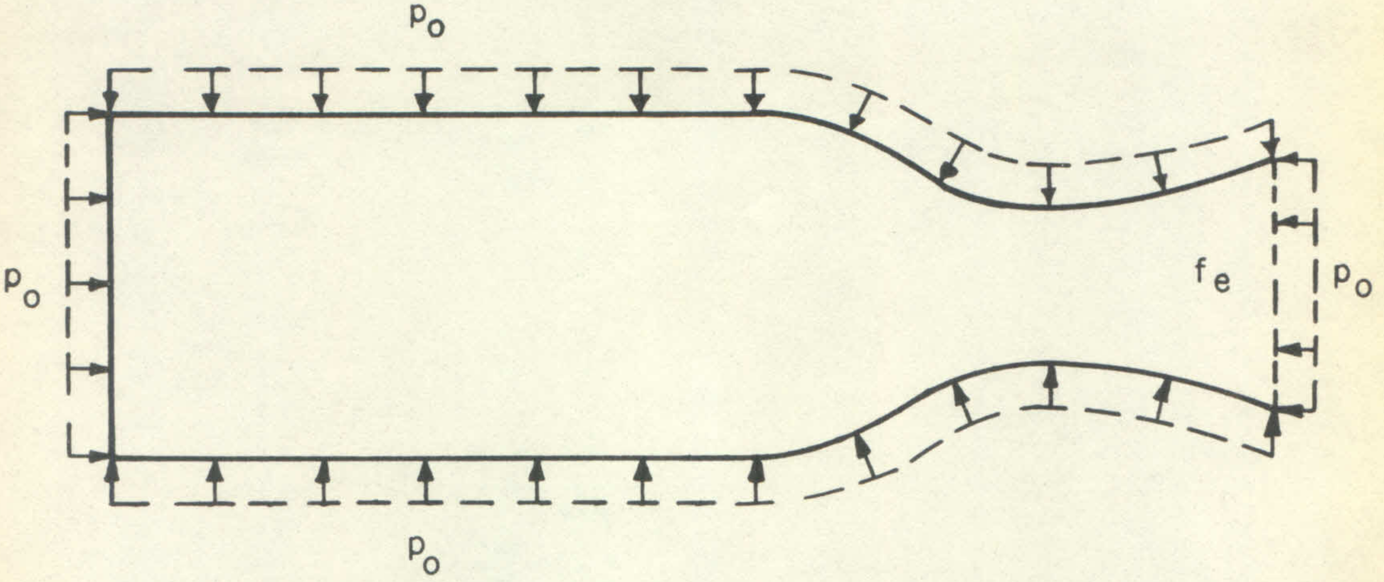
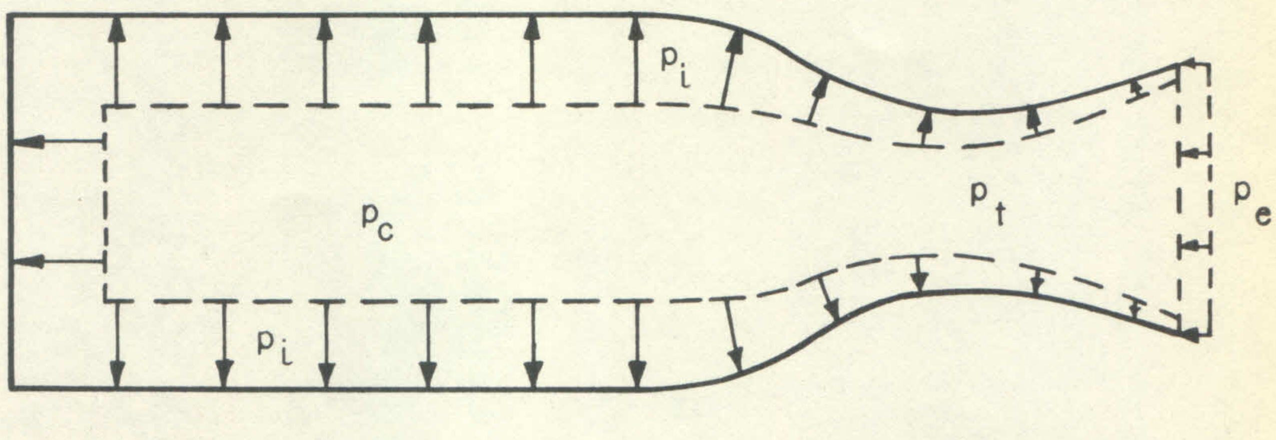

THROAT
CONVERGENT SECTION
DIVERGENT SECTION
$\frac{P}{P_t}$
$\frac{\rho}{\rho_t}$
$\frac{T}{T_t}$
$\frac{v}{v_t}$
3
2
1
0
0
.2
.4
.6
.8
1
.8
.6
.4
.2
0
RATIO OF THROAT AREA TO CROSS-SECTION AREA A_T/A

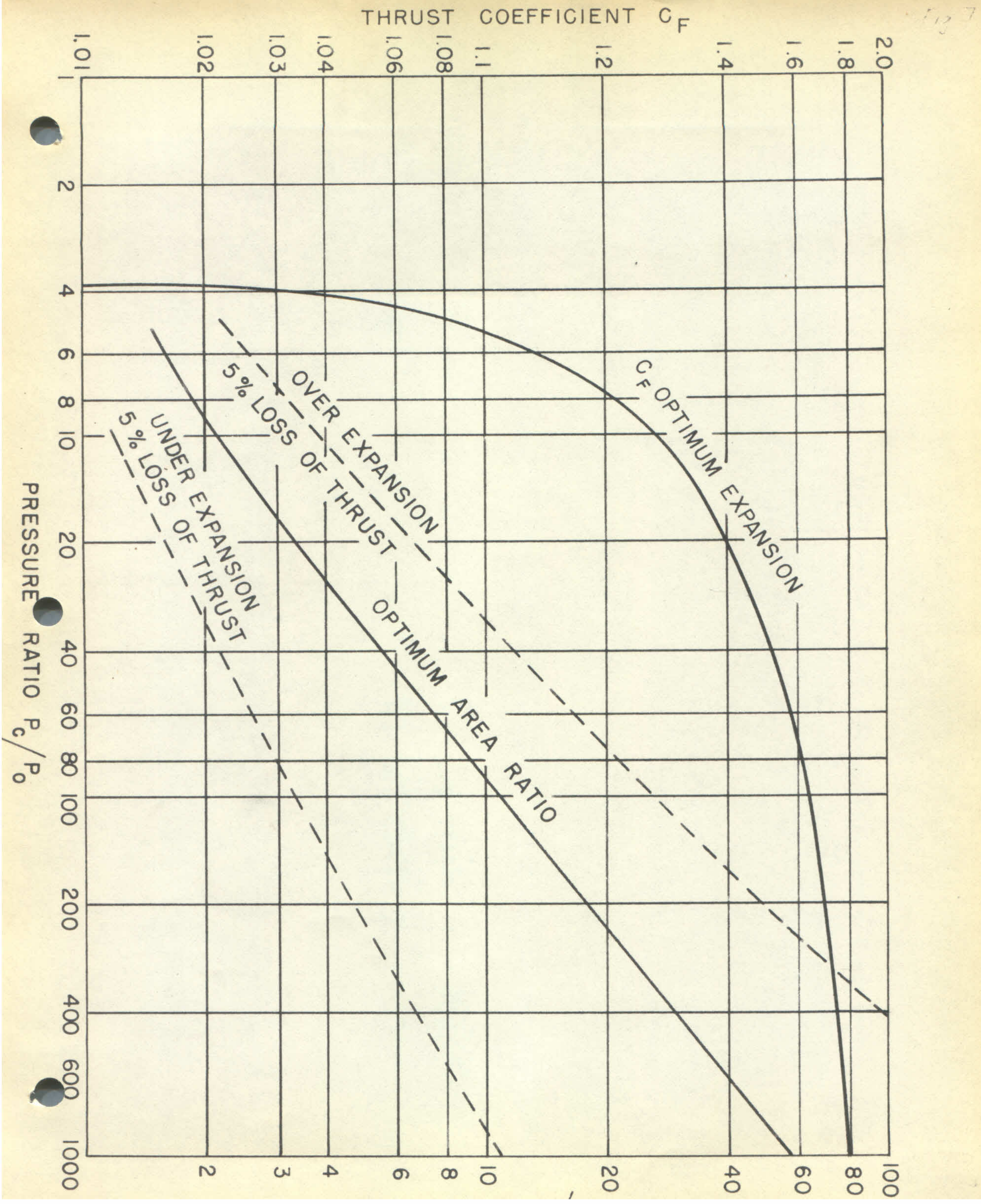
Fig 6



(a)



(b)


THRUST COEFFICIENT C_F
1.01
1.02
1.03
1.04
1.06
1.08
1.1
1.2
1.4
1.6
1.8
2.0
PRESSURE RATIO P_c/P_o
1
2
4
6
8
10
20
40
60
80
100
200
400
600
1000
2
3
4
6
8
10
20
40
60
80
100
C_F OPTIMUM EXPANSION
OVER EXPANSION
5 % LOSS OF THRUST
UNDER EXPANSION
5 % LOSS OF THRUST
OPTIMUM AREA RATIO

PART II. SOLID PROPELLANT ROCKETS

Section A. Characteristics of Solid Propellant Rockets

1. Introduction

To those who were not directly concerned with rockets during the war, the term "rocket" is likely to bring to mind what are more technically described as artillery rockets, that is: rocket propelled missiles. Most artillery rockets are propelled by solid propellant rocket motors. The term "solid propellant arising from the fact that the propellant, before its combustion, has mechanical properties similar to the mechanical properties of a solid body. The much publicized Bazooka ia an artillery rocket of this type. The German V-2 rocket is an artillery rocket, but it is a liquid propellant rocket.

During World War II intensive development of rocket devices was carried on by the major countries taking part in the conflict. Despite the recency of this development effort, rockets, as war weapons, are not new. Early in the nineteenth century Sir William Congreve is credited with developing a variety of successful war rockets. These rockets were used with devastating effect by British soldiers in the siege of Copenhagen in 1807. However, during the second half of the nineteenth century the art of gunnery was so successfully developed that cannon soon were far superior to rockets both in range and accuracy. The question is then, why was there a great effort made in artillery rocket development in the recent war? The answer is that the emphasis placed upon mobility, firepower, and the use of aircraft in the recent war has again made the artillery rocket an important war weapon.

Another development which recently received serious attention was the application of rockets to aircraft, either as boosters or as the sole means of propulsion. Solid propellant rockets, which can efficiently supply a given thrust for only a limited time, are not especially well suited for use as a sole propulsive device. However, solid propellant rockets of a special type were developed which could fill the important need of assisting the take-off of the faster and heavier aircraft which were requiring longer and longer runways. This assisted take-off rocket (called JATO for jet assisted take-off) was especially important in aircraft carrier operation, where the length of runways is necessarily quite restricted.

The artillery rocket is a much more mobile weapon than the gun. The rocket launcher is merely an aiming device, it does not have to confine the propelling gas as does a gun tube, nor does the launcher have to absorb a large recoil. Consequently the launcher for an artillery rocket will weigh about the same as one round of ammunition. This is in strong contrast to the 75-millimeter gun, for example, which weighs 2600 pounds and fires a 14-pound missile. Even though artillery rockets are still inferior to guns, both in range and in accuracy, the more mobile rocket can be used from positions further forward and does not have to fulfill such rigid requirements.

For the first time the infantry can carry with it a weapon, the Bazooka, capable of stopping a tank at moderate range. A medium aircraft can fire a barrage of 5-inch rockets equivalent to a destroyer broadside, and after launching its rockets fly just as well as a sister ship which was never equipped for rockets at all. The problem of absorbing the recoil from a 5-inch gun without damage to the frail aircraft structure is a difficult one, and the reduction in aircraft performance entailed in carrying such a weapon would be considerable. Thus we see why the artillery rocket is so important as a supplementary weapon to the gun in modern mobile warfare.

The fundamental characteristic of solid propellant rockets as compared to liquid propellant rockets is greater simplicity in manufacture, in operation, and in servicing in the field. This simplicity means that in runs of not too long duration, about thirty seconds maximum, a solid propellant rocket will weigh less than a liquid propellant rocket which delivers the same thrust. This simplicity makes the manufacture and use of artillery rockets feasible, and gives solid propellant rockets a definite advantage in application to assisted take-off and to several other rocket propulsion problems. Furthermore, the advantage of simplicity is important in many cases when a "handy" booster of some type is required as a purely auxiliary device.

This gain in simplicity is somewhat offset by a sensitivity of performance of solid propellant rockets to climatic conditions, particularly ambient temperature, an effect usually unimportant for liquid propellant rockets. Another disadvantage is lack of controllability. For artillery rockets the duration is so short, one second or less, that control of the thrust during operation is unimportant. However, for assisted take-off rockets with durations up to thirty seconds control of the rocket to the extent of turning it on, off, and on again is impractical although a device sufficiently simple to be practical can be made to allow shutting off the rocket after it is ignited. Control of the amount of thrust from a given solid propellant rocket during its operation, in a practical manner, is not possible.

Thus we see that the most valuable and important characteristic of the solid propellant rocket is simplicity. In situations which require controlled operation a liquid propellant rocket will be better. Furthermore, any "improvement" of a solid propellant rocket which makes it a complicated mechanical device is usually not desirable. As we shall see, improvements in solid propellant rockets should take the fundamental direction of better propellants and better materials of construction, and improved accuracy in the case of artillery rockets.

2. <u>Manner of Operation</u>

A solid propellant rocket unit consists of a charge of solid propellant within a combustion chamber, and an exhaust nozzle through which the products of combustion escape. The reaction due to the expulsion of the combustion products is the thrust of the rocket. The propellants used in rockets, just as those used in guns, do not explode but instead burn away at a definite rate on those surfaces which are exposed to the hot gas or flame within the combustion chamber. The rate at which the surface of the propellant recedes in a direction normal to itself during burning is designated as the rate of burning and is usually expressed as inches per second. The burning rate depends upon chamber pressure, increasing with higher pressures, but for most solid propellants in use today, the burning rates at a pressure of 2000 lbs. per sq in lie between one and two inches per second. Now the thrust of a rocket motor may be considered as equal to the product of exhaust velocity and mass flow, so that, in order to get a large thrust a large burning surface must be used to obtain a large mass flow. Similarly, to obtain a long duration thrust only a small portion of the charge must burn at a time. Since a given combustion chamber can contain only a limited amount of propellant, the thrust may be made large for a short time by providing a small burning surface. A wide variety of arrangements of solid propellant charges have been used in solid propellant rocket units. Here we have space to discuss only two extreme types, the restricted burning and the unrestricted burning rocket, and we will attempt to give a detailed picture only for the simpler restricted burning unit. Later an attempt will be made to describe in a qualitative way the special problems which arise in the case of the unrestricted burning solid propellant rocket unit. A diagrammatic sketch of the two types of rocket we will discuss is shown in Fig. 8.

Fig. 8

In the restricted burning rocket, the propellant charge is made in the form of a solid right circular cylinder. The cylindrical side surfaces and one end face are restricted from burning by a suitable lining or coating, and burning is allowed to proceed from one end only. This type of rocket is sometimes called end burning or cigarette burning. The duration of thrust obtained from a restricted burning rocket is proportional to the length of the charge and depends upon the chamber pressure and the type of propellant used. The thrust obtained from such a rocket is proportional to the area of the circular burning surface, and depends upon the chamber pressure, the type of propellant used, and the efficacy of the rocket unit design.

In the unrestricted burning rocket the charge is often in the form of a hollow right circular cylinder (tubular charge or tubular grain). This charge is held in place by a suitable grid or trap at the rear or exhaust nozzle end and a few centering support points distributed along its length, but is otherwise uninhibited. (The annular end surfaces are sometimes restricted from burning by inhibiting "washers".) The charge is ignited and allowed to burn on all surfaces except for very small areas at support points. The thrust from such a unit is proportional to the total burning surface and depends upon the chamber pressure, the type of propellant used, and the design of the rocket unit and powder grain. The duration is proportional to the thickness of the cylindrical wall (web thickness) and depends upon the chamber pressure, the type of propellant, and the internal geometry of the combustion chamber and the geometry of the powder grain.

The chamber pressure generated by either type of rocket unit is a function of the ratio of the burning area to the cross-sectional area of the exhaust nozzle throat. This ratio is sometimes called the "area-ratio" or K of the rocket. For unrestricted burning rockets the chamber pressure varies considerably from one end of the grain to the other and depends in a marked degree upon the geometry of the propellant charge and the internal geometry of the combustion chamber.

Typical thrust-time performance curves for the two types of rockets may be seen in Fig. 9. The thin web and large burning

Fig. 9

surface of the unrestricted burning rocket favor designs requiring high thrust and short duration, while for similar reasons the converse is true for the restricted burning rocket.

3. Special Characteristics

There are a number of special characteristics of solid propellant rocket units, a knowledge of which is important in understanding their behavior. These characteristics constitute some of the major limitations of the rocket propellants now in use. A list of these characteristics follows:

(a) Temperature sensitivity

(b) Temperature limits

(c) The combustion limit

(d) The pressure limit

(e) Decomposition on storage

Temperature sensitivity

If we take a number of identical rocket units and condition them by storing at different temperatures before firing, we will find that those stored at high temperatures will operate at higher chamber pressure and thrust than those stored at low temperatures. The duration of the high temperature group will be shorter than that of the low temperature group, but the total impulse of all the units tested will be almost the same. We conclude that temperature has an important effect on the rate processes within the unit, but only a minor effect on the total energy or impulse. This is called temperature sensitivity. A quantitative definition is:

$$a = \frac{1}{p_c} \left(\frac{dp_c}{dT_p} \right) \qquad \text{(at constant area ratio)} \qquad (56)$$

Here a is the temperature sensitivity, p_c is the chamber pressure T_p is the temperature of the propellant charge before firing. For restricted burning ballistite the temperature sensitivity is

0.0058/°F, for unrestricted burning ballistite it is much more, 0.0132/°F. For composite asphalt-potassium-perchlorate propellants developed at the Jet Propulsion Laboratory of the California Institute of Technology[13], and for some special

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[13]These will usually be designated as GALCIT propellants, GALCIT being an abbreviation for Guggenheim Aeronautical Laboratory, California Institute of Technology.

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composite propellants developed by the NDRC (National Defense Research Committee) during the war the temperature sensitivity is only 0.0024/°F. The temperature sensitivity of ballistite is so great that when it is utilized in unrestricted rockets dangerously high pressures above 120°F and poor combustion below 0°F limit the use of such rockets to about the temperature range indicated. The temperature sensitivity of the GALCIT and NDRC propellants is small enough so that it is not usually considered objectionable. In the case of unrestricted burning artillery rockets temperature sensitivity impairs accuracy.

Temperature limits

Special limitations on the temperature range within which a rocket may be used are sometimes introduced by a change of mechanical properties of the propellant with temperature. Certain types of the GALCIT propellants may soften and deform sufficiently when stored at high temperature so that an abnormally large burning surface is exposed. Upon ignition the resulting high chamber pressure may cause failure of the unit. Ballistite may soften sufficiently at high temperatures so that the large pressure gradient along the grain which is present in unrestricted burning units may cause the charge to break up, exposing a large burning surface, and lead to failure of the unit.

At low temperatures certain types of the GALCIT propellants may become so embrittled that the charge will shatter upon ignition of the unit. The large burning surface exposed in a failure of this type may lead to a violent explosion. The NDRC composite propellants do not seem to be subject to temperature limitations due to change of the mechanical properties of the propellant with temperature.

<u>Combustion limit</u>

Let us imagine that we have a number of rockets identical except for the exhaust nozzle throat diameter. Let us fire them starting with the units with the smallest throat and going in order to the next larger throat size. We can then plot a curve of chamber pressure as a function of exhaust nozzle throat diameter, finding, as one might expect, lower pressures with larger nozzle openings. Eventually however, as we exceed a throat of a certain diameter, the chamber pressure will be found to be far below the pressure predicted from the high pressure portion of the curve. If a number of units are fired with this same large throat diameter the chamber pressure will vary erratically from unit to unit. Finally if the exhaust nozzle throat is made large enough individual units will not burn continuously but will burn in an irregular manner with a chugging noise. This last phenomenon is called chuffing. This is illustrated in Fig. 10.

Fig. 10

From this we see that a given propellant cannot be used at an arbitrarily low pressure, and the designer of a rocket unit must design at chamber pressures above the combustion limit if reproducible performance from unit to unit is to be obtained. When low over-all rocket unit weight is desirable, a combustion limit at high pressure is a serious disadvantage since the weight of the walls of the combustion chamber is directly proportional to its volume and to the design chamber pressure.

The combustion limit of ballistite is about 500 lb/sq in of GALCIT propellants is about 1000 lb/sq in. The NDRC composite propellants have a combustion limit not exceeding 100 lb/sq in.

The combustion limit of the straight nitrocellulose propellants frequently used in guns is above 5000 lb/sq in and therefore, such propellants are not suitable for rockets. Ballistite, consisting of roughly equal parts of nitrocellulose and nitroglycerine is essentially a modification of the gun propellants for rocket use.

Pressure limit

Some propellants may safely be used only below a critical chamber pressure. If the critical chamber pressure is exceeded, the propellant burns in a violent and unpredictable manner. Brittle propellants with a granular structure are particularly subject to this effect. Most of the commonly used rocket propellants have been developed to such a degree that the pressure limit lies above 5000 lb/sq in, and therefore is not a problem in rocket unit design.

Decomposition on storage

The double-base[14] propellants (ballistite and related

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[14]The term "double-base" refers to propellants containing the two basic ingredients nitrocellulose and nitroglycerine in contrast with single-base propellants which are chiefly composed of nitrocellulose.

---

materials) slowly decompose with prolonged storage. Their decomposition is autocatalytic and diphenylamine is usually added to neutralize the effect of the initial decomposition products. It is inadvisable to store ballistite at elevated temperatures long periods of time.

The particular group of composite propellants developed by the NDRC consisting of ammonium picrate and sodium nitrate may become soft and mechanically weak due to absorption of moisture by the latter substance from the atmosphere. These propellants must be shipped in moisture-tight containers and must not be exposed to moisture before use.

The GALCIT propellants seem to store indefinitely with no sign of chemical decomposition.

Section B. Theory of Solid Propellant Operation

1. The Rate of Burning Law

The ideal solid propellant rocket should exhibit a "flat-top" thrust-time curve. Upon ignition, the chamber pressure should rise rapidly but smoothly to a steady value, remain constant as the propellant charge burns away, and then fall off as the residual high pressure gas flows from the combustion chamber. For a given rocket, the thrust and chamber pressure are nearly proportional over a wide range of pressures and one may speak of a "flat-top" performance curve with reference to either thrust or chamber pressure.

Irregular pressure-time curves are not desirable, even aside from difficulties which might arise in the application of units delivering varying thrust, because the strength and hence the weight of the combustion chamber must be designed to withstand the peak pressure. The exhaust velocity from a solid propellant rocket is relatively insensitive to chamber pressure while the weight of the unit is directly proportional to the pressure. The over-all weight of most present day rockets could be reduced by a considerable factor if solid propellants with a lower combustion limit were available.

We will now discuss, within very narrow limits, the theory of the combustion process within a rocket unit. The material will be limited entirely to the restricted burning rocket because it is in principle simpler, and later there will be indicated in a qualitative way the special problems of the unrestricted burning rocket. In our discussion we will follow the work of von Karman and Malina[15].

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[15] Th. von Karman and F. J. Malina, restricted report prepared in 1940 for the Army Air Corps.

As we have indicated, very little can be done in a practical way to regulate the operation of a solid propellant rocket once it has been ignited. The parameters controlling the operation of the unit must be designed into it from the start. The rate of burning law describes in a purely empirical way the factors which affect the rate at which the solid propellant burns away in layers parallel to the burning surface. Some efforts have been made to obtain theoretical expressions for the rate of burning law based upon the heat of combustion, the specific heat, and other more fundamental characteristics of the propellant. The combustion of a solid propellant is a very complicated process involving reactions in the solid state, reactions in the liquid phase if any, and reactions in the gas phase. For actual solid propellants the reactions in any one phase are very complicated and take place at high pressure, and the reactions in all the various phases present are mutually interdependent. It has taken years of empirical study to find out the proper theoretical approach to the kinetics of the simple, low pressure, gas phase, hydrogen-oxygen combustion process. Consequently, as one might expect, we find that the theories of the solid propellant burning rate are quite provisional and involve many approximations. Although these theories frequently give valuable insight into the combustion process, they are not sufficiently quantitative to furnish a satisfactory basis for investigation of the internal ballistics of the rocket.

From special experimental studies it has been found that the rate of burning, r, of a solid propellant may be expressed as a function of the following variables:

$$r = r\,(p_c, T_p, v, t) \qquad (57)$$

Here r is the rate of burning, p_c is the chamber pressure, T_p is the temperature of the propellant charge before ignition, v is velocity of gas flow parallel to the burning surface (this is obviously not present in restricted burning rockets), and t is the time measured from the instant of ignition. For the restricted burning rocket the dependence of r on v and t is small and we will use the expression

$$r = ap_c{}^n \qquad (58)$$

where a and n are constants, determined purely experimentally, which depend upon the propellant under consideration. The constant a is assumed to depend upon the temperature, and n is assumed to be independent of temperature. The constant n lies between 0.4 and 0.8 for the solid propellants commonly used in rockets, and a is of such an order of magnitude that r is about one inch per second when p_c is 2000 lb/sq in.

2. Stability of the Shape of the Burning Surface

Consider the schematic restricted burning rocket outlined in Fig. 11. Suppose that the burning surface f_c is not flat

Fig. 11

as indicated but, because of faulty preparation or because of combustion of an inhomogeneous region of propellant charge, it has become concave or of some other irregular shape. Will the burning surface "flatten out" as burning proceeds or will it become more and more distorted leading to an excessively large burning area and ultimate failure of the unit? This question is important because, as we will see, a 10 per cent increase in burning surface can cause a 70 per cent increase in chamber pressure.

If we assume that the burning surface moves normal to itself, and burns away at the same rate at every point, then the sketches in Fig. 12 show that it tends to "flatten out"

Fig. 12

as burning proceeds. The assumption that the surface moves perpendicular to itself at each point can be made plausible from symmetry considerations. For restricted burning rockets the velocity of gas flow is almost zero at all points of the surface and the pressure is nearly uniform, so that from equation (58) we should expect the burning rate to be the same at each point. Unrestricted burning will be considered later on. The general conclusion is that the shape of the burning surface is stable, that is, the surface tends to keep flat as the burning proceeds.

3. The Fundamental Differential Equation

It will be shown presently that an equilibrium chamber pressure exists for solid propellant rockets in the sense that, if the burning surface of the charge remains constant as the charge burns away, then the chamber pressure remains constant during the burning period. Assuming that the burning surface is constant one can write an equation for the conservation of mass as follows:

$$\left\{\begin{matrix}\text{Mass burned}\\ \text{per sec}\end{matrix}\right\} = \left\{\begin{matrix}\text{Increase of mass}\\ \text{of gas in the}\\ \text{combustion chamber}\\ \text{per sec}\end{matrix}\right\} + \left\{\begin{matrix}\text{Mass flow}\\ \text{out through}\\ \text{exhaust nozzle}\\ \text{per sec}\end{matrix}\right\}$$

This equation may be written, with the aid of equation (39) derived for the mass flow from an exhaust nozzle and the definition of burning rate as a velocity, using Figure 11 to make our ideas definite, as:

$$r f_c \rho_p = \frac{d}{dt}(\rho_c V_c) + \Gamma \sqrt{R T_c}\, f_t \rho_c \tag{59}$$

$$\Gamma = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} = \frac{\Gamma'}{\sqrt{\gamma}} \tag{60}$$

Here f_c is the area of the burning surface, assumed to be constant, ρ_c is the density of the gas in the combustion chamber, ρ_p is the density of solid propellant, V_c is the volume of gas in the combustion chamber, which will increase in time as the charge burns away. R is the engineering gas constant equal to the universal gas constant divided by the average molecular weight of the propellant gas, f_t is the area of the exhaust nozzle throat, γ is the ratio of specific heats for the products of combustion, T_c is the temperature of the gas within the combustion chamber, sometimes called the flame temperature, and t is the time measured from the instant of ignition.

Equations (59) and (60) assume that the products of combustion obey the perfect gas law, and that p_c is in excess of the critical exhaust nozzle discharge pressure.

Now if the combustion within the rocket takes place at constant pressure, then the temperature, T_c, will have a constant value given by:

$$c_p T_c = H_p \tag{61}$$

Here c_p is the mean specific heat at constant pressure for the propellant gas, and H_p is the heat of combustion of the solid propellant at constant pressure. We neglect the effect of the initial propellant temperature on T_c. For ballistite T_c is about 5000°F(5460°R) and for other propellants values range from 3000°F to 4000°F. A rather complicated analysis shows that T_c is within 5 per cent of the value given by equation (61) in 10 milliseconds after ignition, and that T_c remains within a few per cent of this value under large pressure fluctuations. Consequently we will assume that T_c is constant with respect to time. With this assumption we have:

$$\frac{d}{dt}(R\, T_c\, \rho_c V_c) = R T_c \frac{d}{dt}(\rho_c V_c) \tag{62}$$

We multiply equation (59) by RT_c, note the equation of state:

$$p_c = R \rho_c T_c \tag{63}$$

and define p_p, a constant for a given propellant, by:

$$p_p = \rho_p R T_c \tag{64}$$

and finally obtain:

$$r f_c\, p_p = \frac{d}{dt}(p_c V_c) + \Gamma \sqrt{R T_c}\, f_t\, p_c \tag{65}$$

where pressures replace the densities of equation (59).

In equation (64) the constant p_p has the dimensions of pressure, and because ρ_p, the density of the solid propellant is much greater than the density of the gas in the combustion chamber, p_p has numerical values of 100,000 to 200,000 lb/sq in. Large values of p_p indicate high specific impulse for the propellant, and to a limited extent p_p represents a merit index for solid propellants.

By the geometry indicated in Fig. 11, and the assumed constancy of f_c we have:

$$\frac{d}{dt}(p_c V_c) = V_c \frac{dp_c}{dt} + p_c \frac{dV_c}{dt} = V_c \frac{dp_c}{dt} + p_c r f_c \tag{66}$$

We can now write equation (65) as:

$$\frac{dp_c}{dt} = \frac{1}{V_c}\left[r(p_p - p_c) - \Gamma \sqrt{R T_c}\left(\frac{f_t}{f_c}\right) p_c \right] \tag{67}$$

And also V_c is given as:

$$V_c = V_c^0 + f_c \int_0^t r\,dt \tag{68}$$

where V_c^0 is the "free" volume within the combustion chamber which is not filled initially with solid propellant.

Equations (67) and (68) can be solved simultaneously by numerical methods, where r as a function of p_c is known, to give the variation of chamber pressure with time.

4. <u>Stability of the Chamber Pressure</u>

We note that when the bracket on the right of equation (67) is zero the chamber pressure does not vary with time. Let us call the corresponding value of the chamber pressure p_s. Now the algebraic sign of dp_c/dt is the same as the sign of the square bracket since V_c is always positive.

Let us plot separately the two terms in the square bracket denoted as follows:

$$m_i(p_c) = r(p_p - p_c) = (p_p - p_c)\, a\, p_c^n \tag{69}$$

$$m_o(p_c) = \Gamma \sqrt{RT_c}\ \left(\frac{f_t}{f_c}\right) p_c \tag{70}$$

Most rocket motors will fail mechanically if their internal pressure exceeds 10,000 lb/sq in., and recalling the magnitude of p_p we may write for (69)

$$m_i(p_c) \overset{o}{=} p_p\, a\, p_c^n \tag{71}$$

Now when m_i is larger than m_o equation (67) shows that p_c is increasing, and when m_i is less than m_o we see that p_c is decreasing. In Figure 13 we have plotted m_i, m_o, for a

Fig. 13

propellant having for the exponent n in the rate of burning law the value $n = 0.76$, a common value, and in Figure 14 we have plotted an example with $n = 2$.

Fig. 14

In Fig. 13 we see that if the chamber pressure exceeds p_s by a small amount, then m_o exceeds m_i and dp_c/dt becomes negative, and the pressure returns to p_s. Similarly if p_c should fall below p_s then m_i is greater than m_o and the pressure rises again to p_s. Thus the chamber pressure is stable against small disturbances.

In Fig. 14, on the other hand, we see that if p_c exceeds p_s, then m_i exceeds m_o and dp_c/dt is positive and the pressure rises still further. Conversely if p_c is less than p_s the pressure eventually falls to zero. Thus if $n = 2$, p_s is a point of unstable equilibrium.

The special cases above hold for all values of the ratio f_c/f_t in the expression for m_o, since as this area ratio is varied one gets a family of straight lines through the origin. From elementary calculus we know that $n > 1$ corresponds to curves of m_i convex towards the p_c axis, and $n < 1$ corresponds to curves concave towards this axis. Consequently our argument holds in general, and we say that the chamber pressure is stable if n, the exponent in the rate of burning law, is less than 1.0, and unstable if n is greater than 1.0. In theory, due to the term $(p_p - p_c)$ in m_1, which we have neglected in our study, the limiting case $n = 1$ is also stable. However, for practical rockets, propellants with a rate of burning exponent n greater than 0.85 are so sensitive to minor variations in preparation as to be unreliable and dangerous.

5. The Equilibrium Chamber Pressure

If we now assume n is less than 0.85, then we know a stable chamber pressure p_s (which we will call p_c from now on to conform to standard notation) exists. This value is given by setting $dp_c/dt = 0$ in equation (67). This gives for the equilibrium chamber pressure:

$$f_c/f_t = \frac{\Gamma \sqrt{RT_c}\; p_c}{r(p_p - p_c)} = \frac{\Gamma \sqrt{RT_c}}{a(p_p - p_c)}\; p_c^{\,1-n} \qquad (72)$$

A typical curve showing the variation of chamber pressure with area ratio f_c/f_t, sometimes called K_n, is shown in Fig. 15, along

Fig. 15

with a rate of burning curve. As we would expect, higher pressures are obtained at larger area ratios, i.e., at smaller exhaust nozzle openings.

For the older propellants the combustion limit is such that the design point for a rocket unit was on a rather steep point of the area ratio curve. An approximate solution of equation (72) for p_c in terms of f_c/f_t shows that p_c varies as f_c/f_t to the power $1/1-n$, and when $n = 0.8$ there results a very steep fifth power law. This illustrates the delicate balance between gas evolution within the rocket motor from the combustion of propellant, and the escape of gas through the exhaust nozzle. One should realize that this is a dynamic balance depending upon a rate process, the rate of burning law, and minor variations in the properties of the propellant can have major effects on the rate of burning and yet cause no detectable change in the heat of combustion, specific gravity, or other properties often used to control the preparation of gun propellants. Consequently special tests are needed to control the preparation of rocket propellants.

Assuming that the area ratio is constant, and neglecting p_c relative to p_p one may differentiate equation (72) with respect to the temperature of the propellant, T_p, (recall a is assumed to be a function of T_p) to obtain:

$$\frac{1}{p_c}\left(\frac{dp_c}{dT_p}\right)_{\substack{\text{constant}\\\text{area}\\\text{ratio}}} = \left(\frac{1}{1-n}\right)\frac{1}{a}\left(\frac{da}{dT_p}\right) \qquad (73)$$

Here again the factor $1/1-n$ is seen as a magnification factor. For one might think of $(1/a)(da/dT_p)$ as a measure of the intrinsic temperature sensitivity of the propellant, but in a rocket, if $n = 0.8$, say, this effect is increased five fold. Some clever chemical modifications of solid propellants are known which will reduce n from 0.75 to 0.45.

We conclude that a solid propellant will be satisfactory as a rocket propellant if the exponent in the rate of burning law lies between 0 and 0.85[16], and that the shape of the burning surface is stable.

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[16]We have said nothing about how quickly the chamber pressure returns to the equilibrium value if it is displaced from this value. A simple approximate solution of equation (67), assuming V_c is constant gives:

$$\Delta p_c(t) = \Delta p_c \text{ (initial instant) } \exp\left[- (p_p/p_s)kt/(V_c/f_c) \right]$$

where k is a constant of value 0.2 in/sec. The recovery time is about 0.2 sec.

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6. Design of a Restricted Burning Rocket

A brief outline of a satisfactory design procedure may help to give an intuitive "feel" for the character of a solid propellant rocket. We assume that the required thrust F, and duration t_b, (t_b is sometimes called the burning time) have been given the designer. He is then supposed to have selected a suitable propellant for which he has an experimental rate of burning curve, and also an experimental[17] area ratio-chamber

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[17]Although our simple theory predicts correctly the general shape of the area ratio curve it is based upon the assumption of the perfect gas law, especially for gas flow from the exhaust nozzle.

Hence an experimental curve is needed.

---

pressure curve which will be similar to the curves in Fig. 15. Knowing the combustion limit for the propellant, the designer selects a pressure slightly above the combustion limit pressure. Low pressures are desirable from the standpoint of weight and safety. From experimental data he knows the specific impulse $I_{sp}(p_c)$ of the propellant at this design pressure p_c.

The weight of the propellant is then:

$$W_p = Ft_b/I_{sp} \tag{74}$$

From the experimental rate of burning curve (Cf. Fig. 15) the designer also knows $r(p_c)$, so that the designer knows the length of the propellant charge l_p from:

$$l_p = r\,t_b \tag{75}$$

Knowing the density of the solid propellant, ρ_p, he calculates the end area, f_c, and the diameter of the charge d_c, from:

$$l_p\, f_c\, \rho_p = W_p \tag{76}$$

and

$$\pi/4d_c^2 = f_c \tag{77}$$

From the design pressure, p_c, the designer knows the area ratio required to give this pressure (Cf. Fig. 15), call this $K(p_c)$, then:

$$f_t = f_c(1/K) \tag{78}$$

and

$$\pi/4d_t^2 = f_t \tag{79}$$

The remainder of the problem has to do with design of the exhaust nozzle which has been outlined in Part I, and the design of metal parts. Special design problems also arise because of the high temperature of the products of combustion, but these cannot be discussed here. A typical JATO unit is illustrated in Fig. 16.

Fig. 16

Section C. Special Problems of Unrestricted Burning Rockets

The propellant within an unrestricted burning rocket burns over its whole surface; and, therefore, the products of combustion must escape by flowing past the burning charge. It is clear that the chamber pressure must vary along the length of the charge, being highest at the forward end, and low at the rear or exhaust nozzle end of the charge. The mean velocity of gas flow parallel to the charge will be zero at the forward end and increase as it nears the exhaust nozzle. This is outlined in Fig. 17. In

Fig. 17

addition, as the charge burns away, the general pressure level within the rocket decreases since more space is available for gas passage along the grain. The problem is no longer a steady state problem.

For well designed rockets the burning rate is almost the same at every point on the charge surface because, although higher pressure makes the charge burn faster at the forward end, the higher velocity of gas flow at the rear accelerates the burning rate to almost balance the effect of pressure. Clearly to design a rocket well in this sense is difficult. Good design is necessary since an unevenly burned charge will break up and be ejected as unburned slivers before its full impulse has been delivered to the rocket. Even good rockets may lose 5 per cent of their charge in this manner.

A long range artillery rocket should be designed with a small diameter to reduce drag. But to put a given amount of propellant in a long narrow tube means that the charge will tend to "choke off" the gas flowing along the grain due to narrow wall and bore clearance. If this "choking" is so great as to make the gas flow reach its sonic velocity at some point of the burning charge the rocket is almost certain to function badly if not explode. Thus internal and external ballistics are directly opposed to one another in their requirements.

Finally, since the weight of a pressure vessel is proportional to its volume and its design pressure, it is desirable to put as much charge as possible into a given combustion chamber. The choking effect encountered from adding too much charge limits the volumetric efficiency (ratio of propellant volume to total volume) of the rocket to something less than 75 per cent.

The effects of temperature sensitivity are greatly aggrevated by the geometry of the unrestricted burning rocket, and although clever geometric design of the charge can reduce this effect, the restricted burning rocket seems to be less temperature sensitive than an unrestricted burning rocket utilizing the same propellant.

In general the important problem for solid propellant rockets of all sorts is a deeper understanding of the burning mechanism of solid propellants so that the burning rate may be controlled to improve the geometry of rocket design and reduce temperature sensitivity.

Section D. Solid Propellant Chemicals

Solid propellants for rockets are crudely classified as homogeneous and composite. The most important homogeneous materials are ballistites consisting of roughly equal parts of nitrocellulose and nitroglycerine. Composite materials include two types of propellants developed by the NDRC, one consists of ballistite modified by the addition of large amounts of inorganic salts to reduce the temperature sensitivity of the propellant, the other consists of various modifications of a mixture of equal parts of pulverulent ammonium picrate and sodium nitrate molded together under high pressure with about 10 per cent of an artificial resin binder. The early GALCIT propellants consisted of 75 per cent pulverulent potassium perchlorate (oxidizer) mixed with 25 per cent asphalt (fuel). This material was mixed and poured into the combustion chamber of a rocket while hot, and then cooled to a tough mass something like paving tar. General characteristics of the propellants are listed in Table II.

TABLE II

General Characteristics of Solid Propellants

Propellant	Flame Temperature	Exhaust Velocity ft/sec	Specific Impulse Sec	Burning Rate At 1500 lb/sq in	Temperature Sensitivity	Density lb/cu ft
Ballistite	5000°F	7000	200	0.7 in/sec	high	100
*NDRC	3000 - 4000°F	5500	180	0.2 - 1.0 in/sec	low	115
*GALCIT	3000 - 4000°F	5500	180	1.4 in/sec	low	115

*Copious quantities of smoke in the exhaust jet, a disadvantage in some applications.

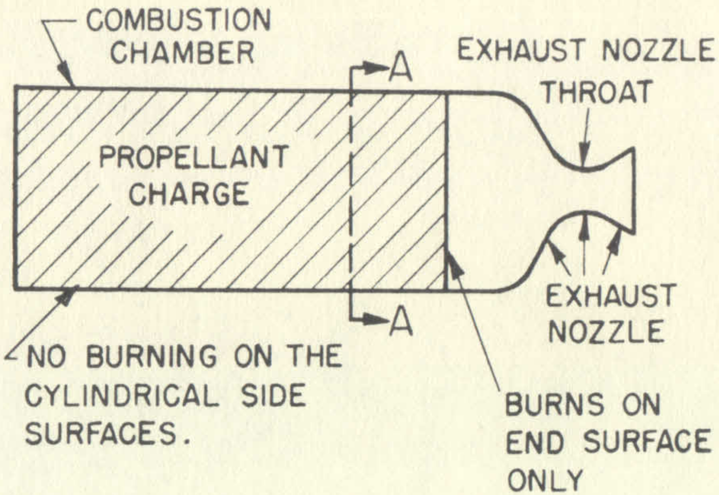


FIG. 2.1a. THE RESTRICTED BURNING SOLID PROPELLANT ROCKET MOTOR.
(SECTION A-A IS CIRCULAR)

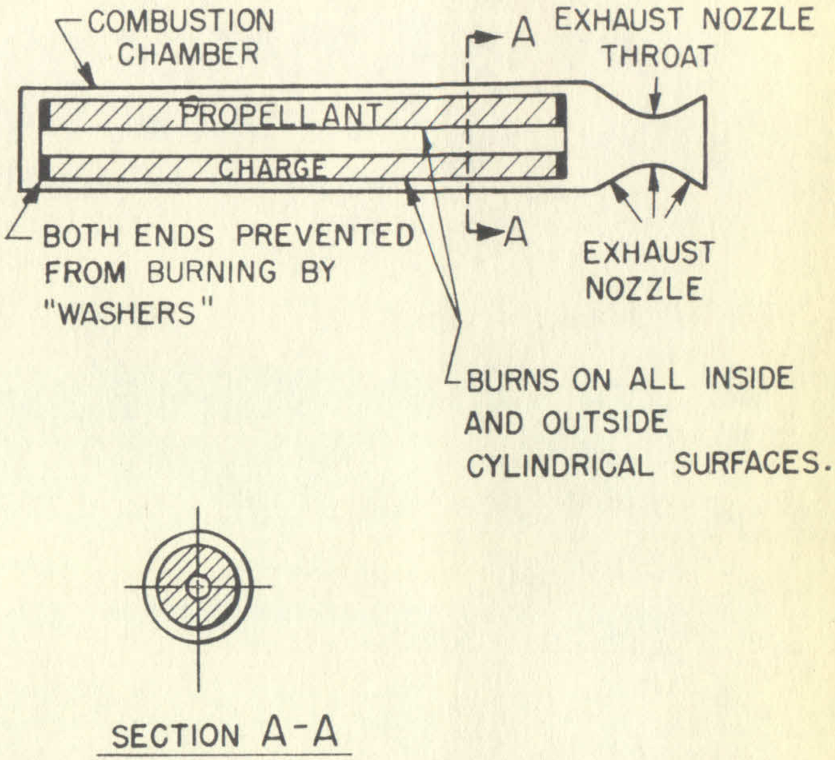
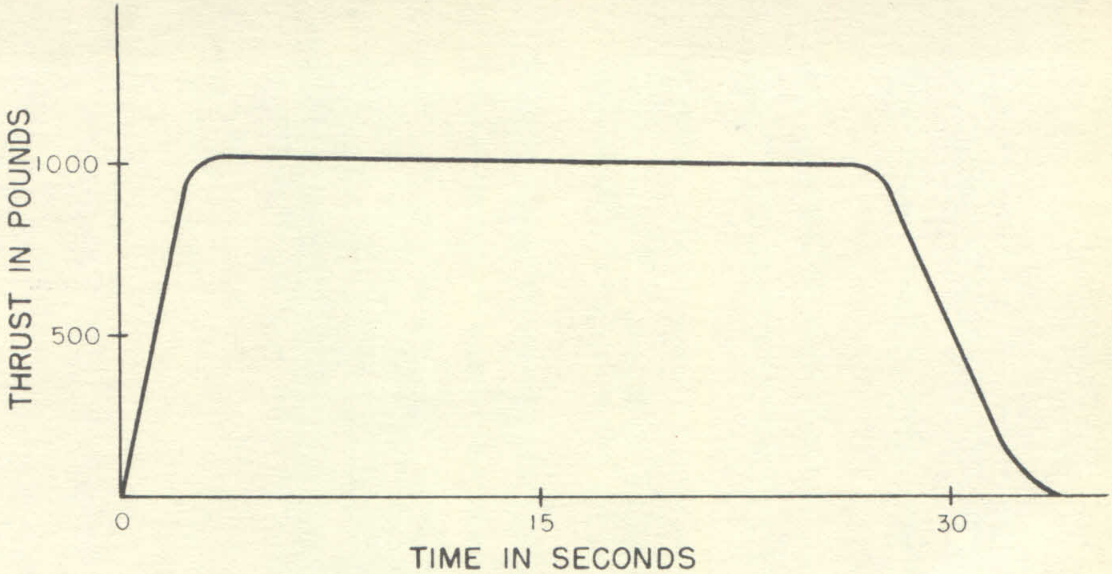


FIG. 2.1b. THE UNRESTRICTED BURNING SOLID PROPELLANT ROCKET UNIT.

FIG. 8 TWO TYPES OF SOLID PROPELLANT ROCKET UNIT

TYPICAL THRUST - TIME PERFORMANCE CURVES FOR THE TWO TYPES OF ROCKETS MAY BE SEEN IN FIG. 9 THE THIN WEB AND LARGE ROCKET FAVOR DESIGNS REQUIRING HIGH THRUST AND SHORT DURATION, WHILE FOR SIMILAR REASONS THE CONVERSE IS TRUE FOR THE RESTRICTED BURNING ROCKET.



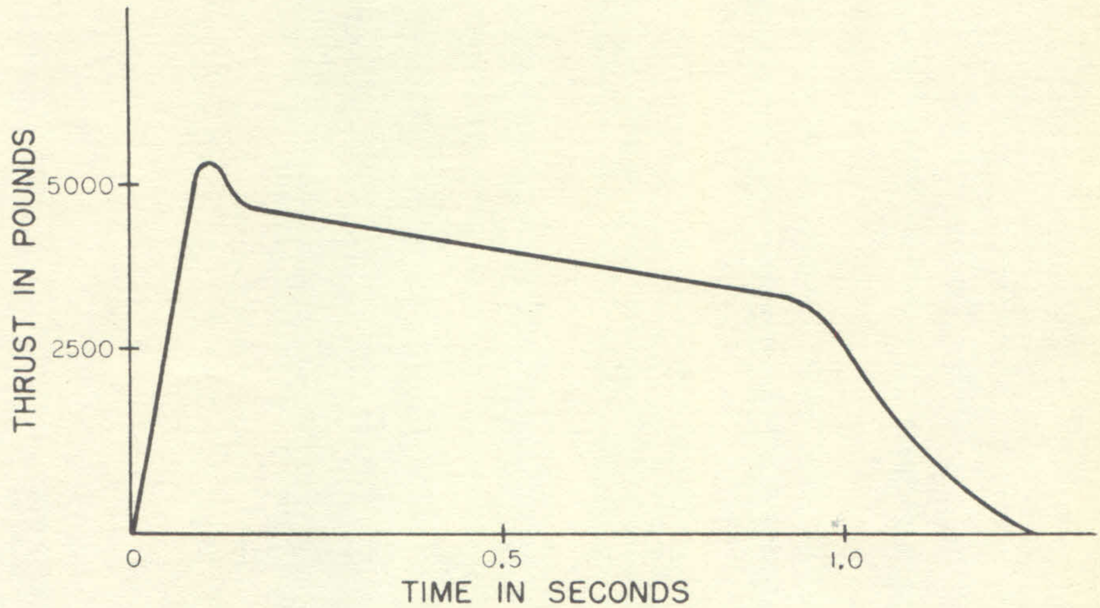


FIG. 9 TYPICAL THRUST - TIME CURVES FOR A RESTRICTED BURNING ROCKET (ABOVE) AND AN UNRESTRICTED BURNING ROCKET (BELOW). NOTE "IGNITION PEAK" IN THE LATTER.

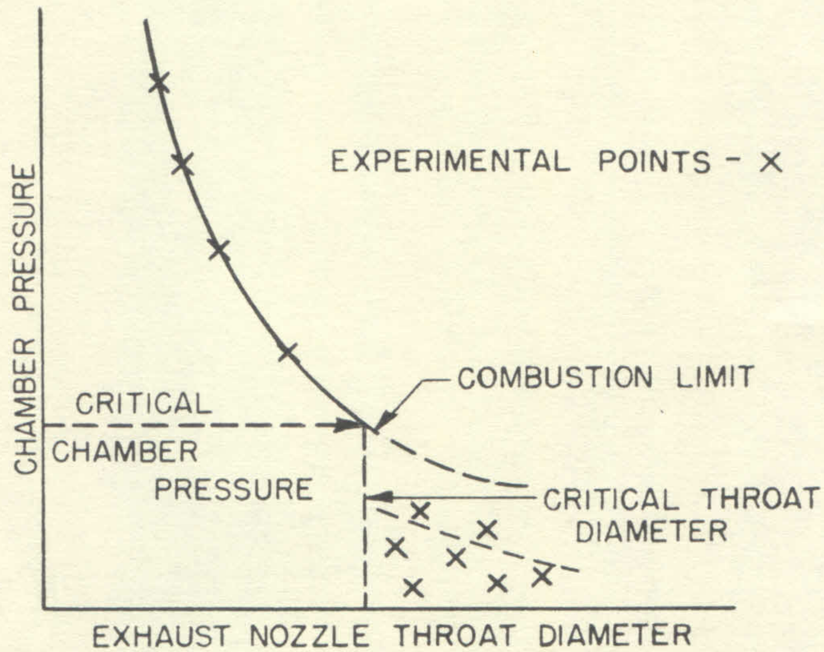


FIG. 10 ILLUSTRATING THE COMBUSTION LIMIT

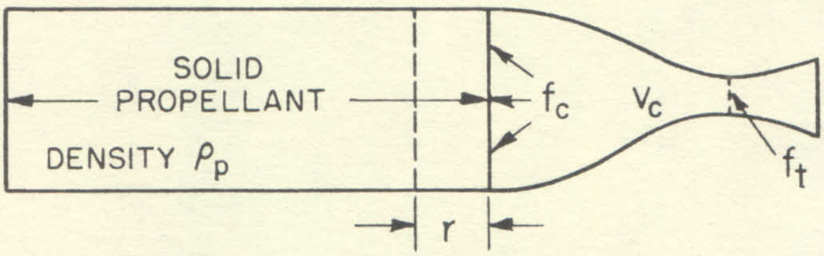


FIG. 11 RESTRICTED BURNING SOLID PROPELLANT ROCKET SHOWING NOTATION USED IN DISCUSSION OF STABILITY.

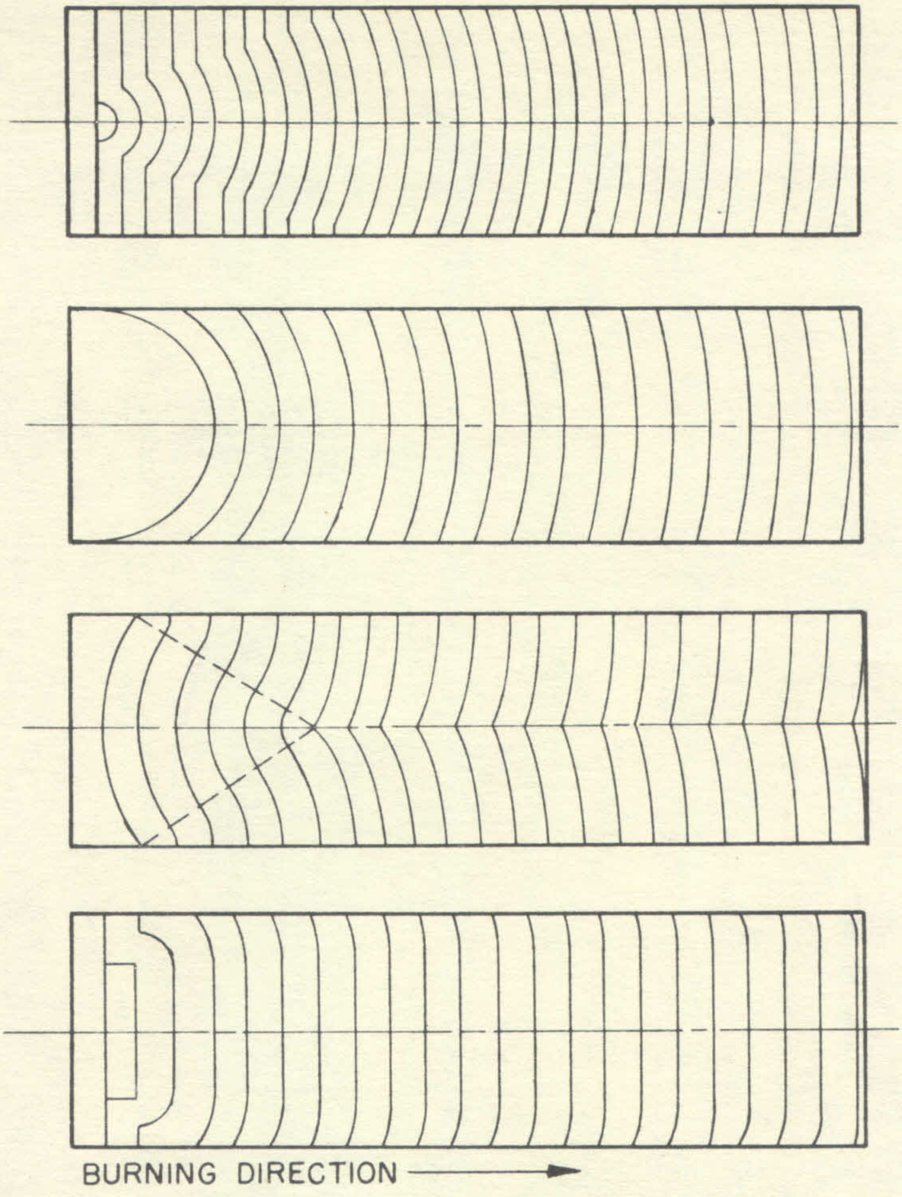


FIG. 12 SKETCHES ILLUSTRATING BURNING SURFACE STABILITY.

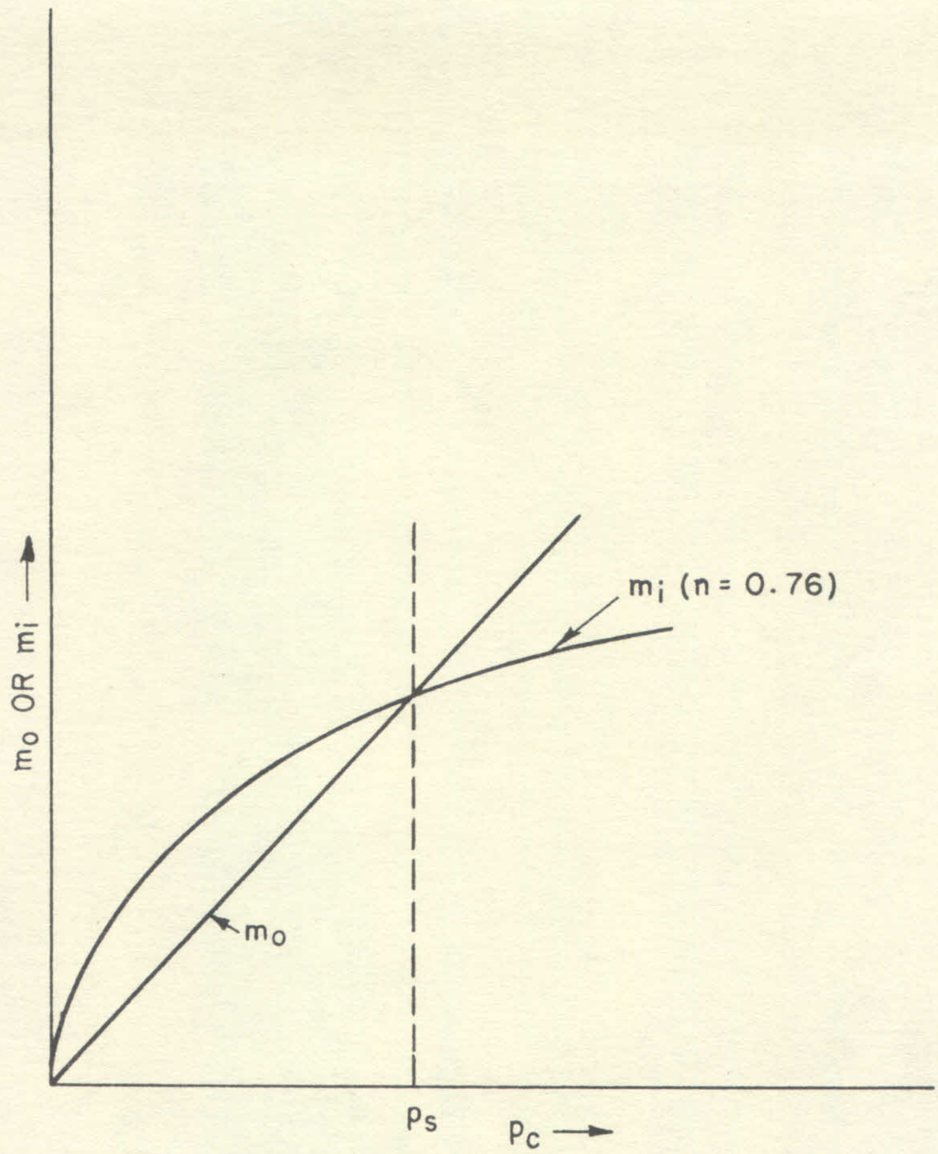


FIG. 13 CHAMBER PRESSURE STABILITY CURVES FOR n = 0.76 .

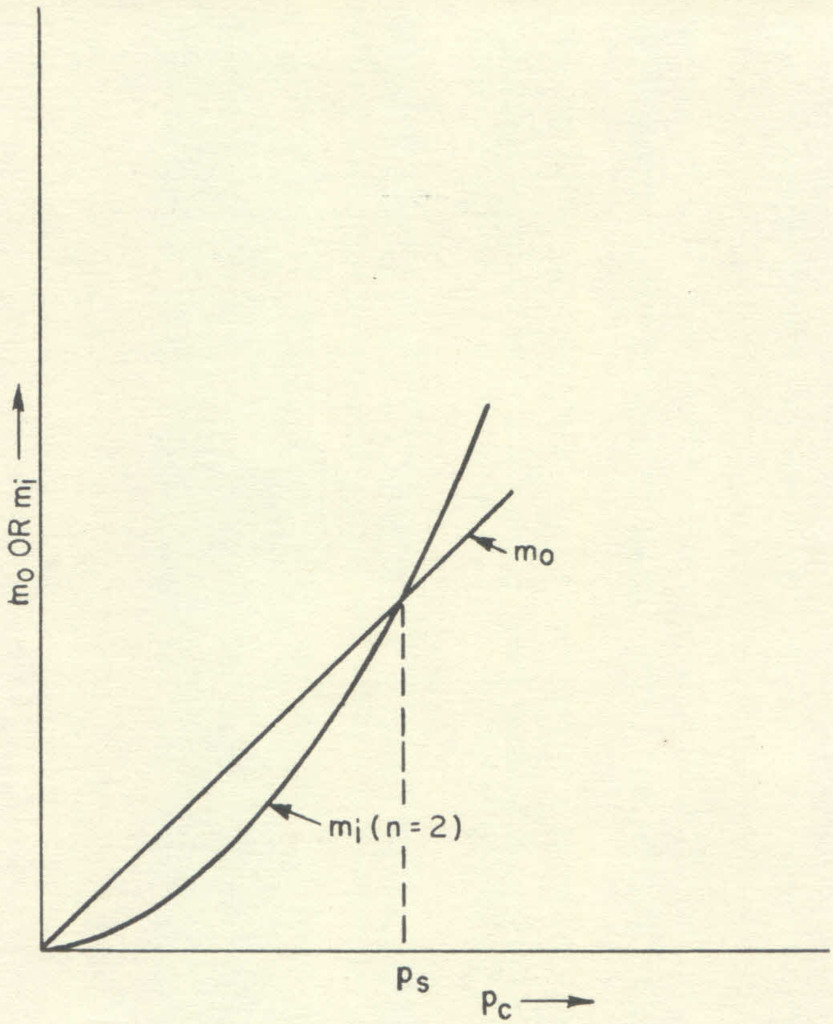


FIG. 14 CHAMBER PRESSURE STABILITY CURVES FOR n = 2 .

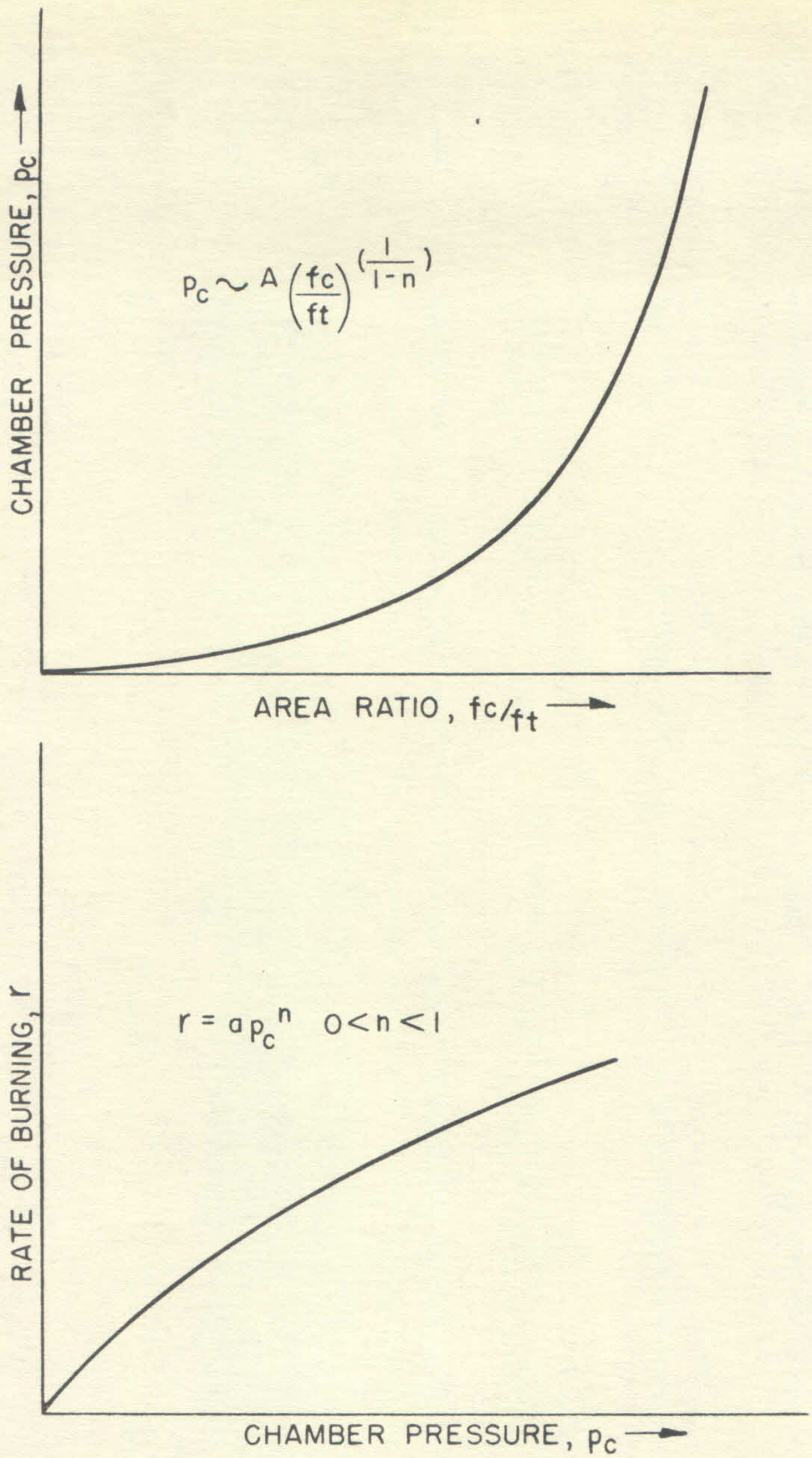


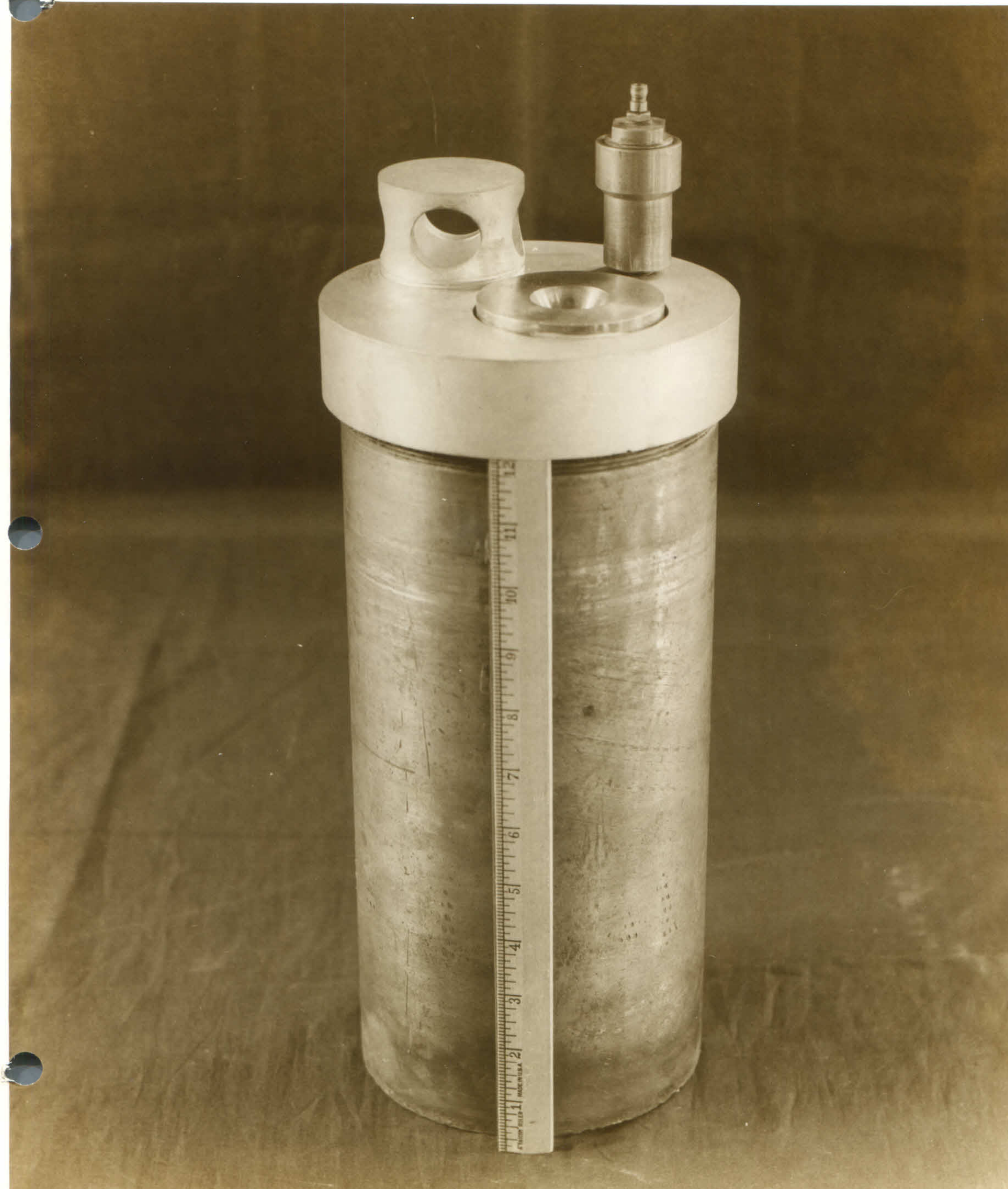
FIG. 15 AREA RATIO EQUILIBRIUM CHAMBER PRESSURE CURVE, AND RATE OF BURNING CURVE.

Fig. 16

Typical JATO Unit

(Photo)

Legend: A typical solid propellant rocket for assisted take-off of aircraft, showing the nozzle, ignitor, and safety rupture disc.



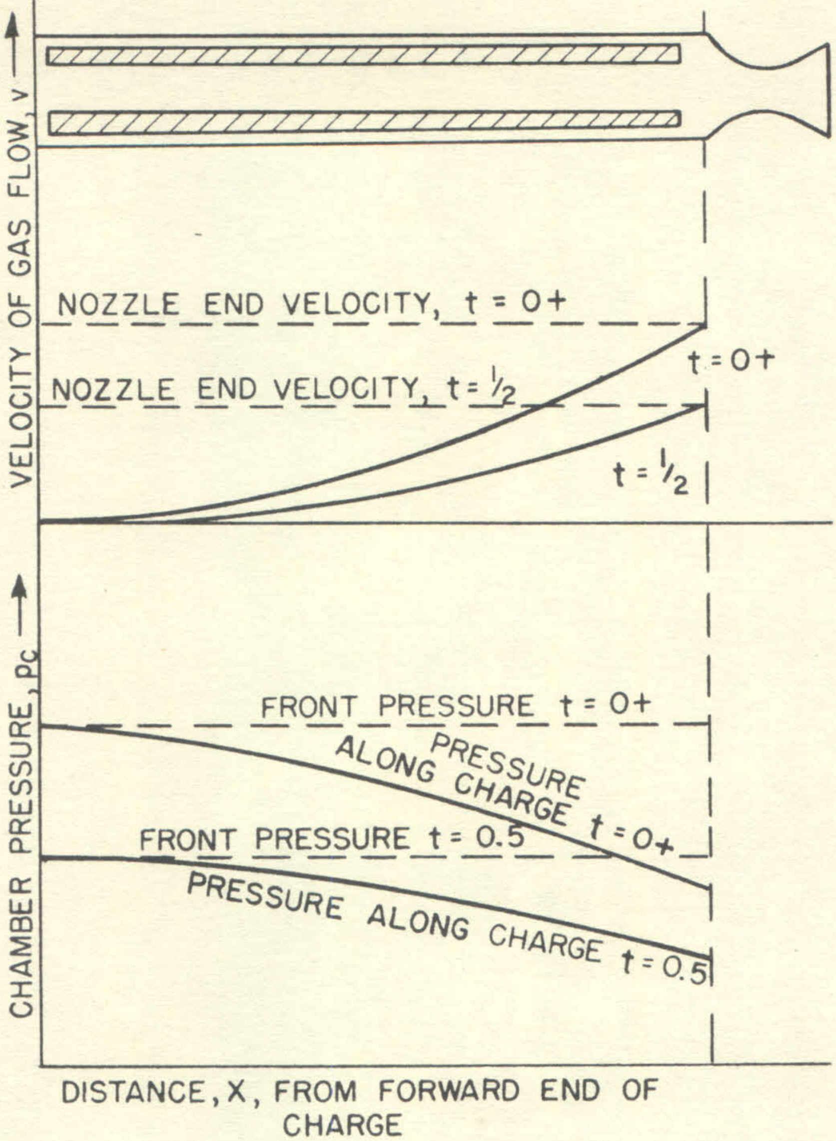


FIG. 17 GENERAL NATURE OF VELOCITY AND PRESSURE DISTRIBUTIONS WITHIN AN UNRESTRICTED BURNING SOLID PROPELLANT ROCKET: SHORTLY AFTER STARTING, AND WHEN THE BURNING IS HALF COMPLETED.

Part III. LIQUID PROPELLANT ROCKETS

Section A. Liquid Propellant Chemicals for Rockets

1. Advantages of Liquid Propellants

A rocket propellant which is liquid rather than solid has two principal advantages. First, most of the propellant may be carried in a light-weight low-pressure vessel and pumped into the combustion chamber, which, though it must withstand high pressures and temperatures, need only be large enough to ensure combustion. This results in a significant weight saving. Second, the flow of liquid may be regulated at will, whereas a solid propellant motor with its entire charge in a single high-pressure enclosure, cannot easily be stopped when it is once ignited. A liquid propellant system has the disadvantage, of course, of being more complicated.

2. Important Properties of Liquid Propellants

Because the nature of the liquid used affects the design of a rocket motor, it is appropriate to review the characteristics of typical propellants. Any liquid combination which will release energy at a high but controllable rate immediately after entry into the combustion chamber may be used as a propellant. However, a reaction which proceeds as a detonation is not admissible in the "constant pressure" process we are considering. For best motor performance the propellant should release a maximum of energy into products of combustion having minimum average molecular weight M and ratio of specific heats γ. In addition to these basic requirements, an acceptable propellant is hedged about with so many practical restrictions that the search for suitable liquids is one of the largest problems of rocket research, and the curious properties of these liquids account for a large portion of the difficulties of rocket motor development.

A tabulation is given below of the significant properties of a propellant liquid, together with brief comments.

a) Heat of Combustion This should be a maximum in order to secure maximum chamber temperature.

b) Molecular Weight of Combustion Products This should be a minimum to secure maximum exhaust velocities.

c) Stability to Shock and Temperature The liquid should not decompose or detonate under mechanical impact or moderately high temperatures.

d) Rate of Reaction This should be high to keep the volume of the combustion chamber small.

e) Ease of Ignition The propellant components should ignite within a minimum time interval after contact with each other or the ignitor flame.

f) Density The higher density liquids require less tank structure and are easier to pump. Smaller tanks reduce air drag.

g) Vapor Pressure This should be low to avoid loss of fluid and the necessity for elaborate thermal insulation, as well as to improve pumping properties.

h) <u>Specific Heat and Thermal Conductivity</u> These should be high for liquids which must serve for coolants as well as propellants.

i) <u>Freezing point</u> This should be low for liquids which may be used in any geographical location.

Other properties of practical importance need no comment:

j) Corrosivity

k) Toxicity

l) Inflammability (especially in vapor phase)

m) Availability

n) Cost

3. <u>Tabulation of Typical Characteristics</u>

A certain rather limited number of liquids have been found which are satisfactory in most of the above qualities. None of them are ideal, and the search for still better substances continues. Some propellants consist of a single liquid; these are termed monopropellants. Those involving two liquids are called bipropellants. Bipropellants normally consist of an oxidizer and a fuel. The ratio of oxidizer mass per unit fuel mass is designated as the <u>mixture ratio</u>. The mixture ratio sometimes deviates from the stoichiometric value in order that lower reaction temperature or molecular weight of products be secured. Below are listed ten typical propellants classified by the nature of the oxidizer.

Bipropellants

Oxidizer		Fuel
Liquid Oxygen	-	Ethyl Alcohol plus Water
Liquid Oxygen	-	Ammonia
Liquid Oxygen	-	Hydrazine
Liquid Oxygen	-	Hydrogen
Nitric Acid	-	Aniline
Nitric Acid	-	Furfural Alcohol
Hydrogen Peroxide	-	Nitromethane
Hydrogen Peroxide	-	C-Stoff (German)

Monopropellants

Hydrogen Peroxide

Nitromethane

Each propellant has its unique characteristics, some of which will now be described. Quantitative performance data are given in Table III.

Liquid Oxygen-Ethyl Alcohol

This classical combination has the advantage of high specific impulse, and in addition its components are non-toxic, non-corrosive, and non-detonable. However, liquid oxygen has a high vapor pressure, which makes it difficult to store and of little value as a coolant. In the V-2 the Germans used a fuel mixture of 75 per cent ethyl alcohol (C_2H_5OH) and 25 per cent water. This water "ballast" decreases both the reaction temperature and average molecular weight. Thus the cooling of the motor is rendered easier without materially reducing its performance.

Liquid Oxygen - Non-Carbonaceous Fuel

The elimination of carbon from the fuel component tends to reduce the molecular weight of the exhaust gases, since the fuel may then be composed largely of nitrogen and hydrogen. For example ammonia, NH_3, is a readily available substance which calculations indicate should give high performance. It is toxic and must be confined under pressure in order to remain liquid at ordinary temperatures. A more convenient fuel is hydrazine, N_2H_4, which is a liquid at ambient temperatures and gives high performance at remarkably low combustion temperatures. (See Table III.) Neither of the above liquids is a very good coolant; the ammonia because of its low boiling point, and the hydrazine because it decomposes at temperatures above a few hundred degrees Fahrenheit.

Liquid Hydrogen

Liquid hydrogen gives the highest performance of any available fuel in combination with liquid oxygen. However its extreme volatility (boiling point approximately 20°K or 38°F degrees above absolute zero) and low density (specific gravity 0.07) combined with its relative scarcity, have prevented its exploitation so far. It is even less suitable as as a coolant than liquid oxygen. However it remains interesting to rocket research workers, representing a sort of ultimate performance goal which they hope to achieve.

Nitric Acid - Aniline

These components have the virtue of igniting spontaneously upon contact, thus eliminating the necessity for special ignition devices. Fig. 18 illustrates the nature of this spontaneous ignition. In order to make ignition more prompt, 6 per cent

Fig. 18.

to 14 per cent of nitrogen dioxide (NO_2) is dissolved in the acid (HNO_3) to make "red fuming" nitric acid, and the water content is kept below a maximum of 2 or 3 per cent. The corrosive acid must be handled in stainless steel or aluminum containers, and its vapor is toxic. These disadvantages are partially offset by the high density (specific gravity 1.55) of the acid and its stability to shock and temperature. Considerable experience in using acid in rocket motors has been accumulated in the United States, the original development being carried out at the Jet Propulsion Laboratory, GALCIT[18].

[18] GALCIT is the abbreviation for Guggenheim Aeronautical Laboratory, California Institute of Technology.

The aniline ($C_6H_5NH_2$) component usually has about 20 per cent of furfural alcohol added to it to depress the freezing point. Aniline is toxic but not otherwise difficult to handle. It has a high boiling point and makes an excellent coolant fluid.

Nitric Acid - Furfural Alcohol

This combination is also spontaneously igniting, with rather less ignition lag than in the acid-aniline case. Here the acid need not contain NO_2 (so-called "white acid"). The furfural alcohol ($C_4H_3O.CH_2OH$) has a low freezing point and is non-toxic. This combination gives equivalent performance and is somewhat more convenient to pump because the lower vapor pressure of white acid as compared with red fuming acid reduces cavitation difficulties.

Hydrogen Peroxide

Hydrogen peroxide of 80 per cent to 90 per cent concentration is an excellant oxidizer, but it rather unstable to heating and very sensitive to contaminants, especially to the metal oxides, which act as catalysts for the exothermic decomposition

$$H_2O_2 \rightarrow H_2O + \tfrac{1}{2}O_2$$

Concentrated peroxide decomposes with explosive violence at temperatures only a little above the boiling point of water. The freezing point ranges from $-10^\circ F$ to $+10^\circ F$ as the concentration increases from 80 per cent to 90 per cent. It may be stored for extended periods provided that it is pure to within a few parts per million, and kept in correspondingly clean containers. Pure aluminum containers are best, although nickel, stainless steel, and certain vinyl plastics are acceptable.

The controlled decomposition of H_2O_2 is usually effected by adding to it a few per cent of concentrated calcium permanganate catalyst solution, or by passing it over solid stones containing permanganate or lead compounds. When decomposed in this manner it serves as a low-temperature rocket propellant providing half the specific impulse at about one-fourth the reaction temperature of other standard propellants. It also may serve as a convenient gas generator for turbine-driven pumps.

Nitromethane

Nitromethane (CH_3NO_2), the simplest of the nitroparaffins, contains all of the elements necessary for combustion in the same molecule. It may therefore be used as a "monopropellant". It must be ignited in the presence of oxygen and will burn smoothly provided a catalyst is added, combustion pressure is about twice the typical value of 300 lbs/sq in absolute, and available combustion volume is several times[19] the value used with other propellants.

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[19] For CH_3NO_2, $p_c = 550$ lbs/sq in absolute and $L^* = 350$ in are typical values. See definition of L* in Part III, Section B.

Nitromethane is non-corrosive, non-toxic, insensitive to contamination, and has a low vapor pressure. The fact that only one fluid need be used makes possible a simple tank and valve system. However, it decomposes with explosive rapidity above

550°F and can be made to detonate under mechanical shock. Consequently it can be used as a coolant only with caution.

When nitromethane is used in conjunction with H_2O_2 as a bipropellant, it will react at lower pressures (300 lb/sq in absolute) and volumes ($L^* = 100$ in) than when used alone. Furthermore, it is now not necessary to add a catalyst to the nitromethane, and ignition can easily be accomplished without a flame or spark, by initiating the peroxide decomposition with a small amount of permanganate catalyst. This combination is a rather convenient propellant because of the non-toxic non-corrosive nature of its constituents, and should be useful in situations where the components do not need to be pre-heated excessively.

C-Stoff - (German)

In their Me163B rocket propelled fighter airplane, the Germans used a fuel designated as "C-Stoff" which contained 30 per cent hydrazine hydrate ($N_2H_4 \cdot H_2O$), 57 per cent methyl alcohol and 13 per cent water. It ignites spontaneously with H_2O_2 by virtue of the hydrazine hydrate content, and is stable enough to be used for cooling the rocket motor.

4. Upper Limit of Chemical Propellant Performance

A comparison of the performance and characteristics of the various propellants discussed is given in Table III. It is seen that, in spite of the great variation in propellants, the difference in performance is not more than 50 per cent. It is sometimes hoped the performance of a rocket can be greatly improved by the discovery of a new propellant. However, thermodynamic calculations based on the known properties of possible liquid propellants set a theoretical upper limit on specific impulse about 80 per cent greater than the best now obtainable. The limitation is fundamental, and follows from the fact that any propellant consisting of the elements H, C, O, and N produces combustion products which dissociate and absorb energy so rapidly above 4000°F that the combustion temperature is definitely limited to values less than 6000°F. Since the limitation is imposed by the nature of the products of combustion, variations in the chemical composition of the propellants cannot produce remarkable increases in specific impulse. The theoretical maximum specific impulse to be expected from liquid oxygen and hydrazine, to mention one little-explored bipropellant, would be about 260 lb sec/lb. Liquid hydrogen combined with either liquid oxygen or liquid flourine would give a specific impulse I_{sp} of about 350 lb sec/lb, which may be considered as the chemical ceiling of performance. Since liquid hydrogen has a specific gravity of only 0.07, it decreases the impulse per unit <u>volume</u> of any bipropellant of which it is a component.

The gain of 25 to 50 per cent in I_{sp} to be anticipated in the near future is well worth working for. But miraculous chemical propellants which increase I_{sp} by a factor of ten over present values do not appear on the horizon. We may conclude then that the choice of propellant in the future will be decided more by utilization factors than by performance.

With the advent of nuclear energy, this conclusion must be modified. However, if it is still necessary to have a working fluid, and if the amount of heat that can be introduced into this fluid is limited by some easily handled temperature it would not be possible to increase the I_{sp} by as much as an order of magnitude over current experimental values. See the discussion of "Problems in the Utilization of Nuclear Energy" in Part IV., Section B. For example, H_2 heated to 4000°K (7200°F) and expanded by the normal adiabatic process yields an I_{sp} = 700 lb sec/lb. If the H_2 were dissociated to 2 H, the value of I_{sp} might increase to approximately 1000 lb sec/lb. These results could be too pessimistic in that other new methods of obtaining a high speed jet may be possible. However, the problem of adapting nuclear energy to rocket propulsion is a difficult one and will require years of research and development.

TABLE III

CALCULATED PERFORMANCE OF TYPICAL LIQUID PROPELLANTS

Propellant	Specific Impulse Isp (sec)	Exhaust Velocity c (ft/sec)	Characteristic Velocity c* (ft/sec)	Mixture Ratio	Chamber Temp. T_c (^oF)
Liquid Oxygen 75% Alcohol - 25% Water	239	7698	5537	1.3	5079
Liquid Oxygen Hydrazine	246	7915	5610	0.33	3632
Liquid Oxygen Ammonia	255	8220	5840	1.4	4951
Liquid Oxygen Liquid Hydrogen	358	11550	8345	3.0	4290
Hydrogen Peroxide (87%)	126	4065	2940	---	1216
Hydrogen Peroxide (87%) Nitromethane	229	7386	5313	0.5	4687
Hydrogen Peroxide (87%) "C-Stoff"	215	6921	4945	2.5	3711
Red Fuming Nitric Acid Aniline	221	7091	5015	3.0	5065
White Nitric Acid Furfural Alcohol	214	6885	4982	1.9	4750
Nitromethane	218	7020	5020	---	3950

NOTE: All values of exhaust velocity c calculated for 300 lbs/sq in combustion pressure, correctly expanded to 1 atmosphere. Subtract 10% to get experimental c, c* or I_{sp} from calculated values.

Section B. Fundamentals of Liquid Propellant Rocket Motors

1. Mechanical Design

A typical liquid propellant thrust-producing component - which shall henceforth be designated a motor - is illustrated in Fig. 19 and Fig. 20, in which the three major parts are in-

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Fig. 19 and Fig. 20

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dicated. These parts are the propellant injector, the combustion chamber, and the exhaust nozzle. The motor shown was designed for use in the high-altitude sounding rocket known as the WAC Corporal, and has the following specifications:

Performance Specifications (Sea Level)

Thrust F	1500 lbs
Combustion Pressure p_c	300 lbs/sq in
Specific Impulse I_{sp}	193 sec
Exhaust Velocity c	6200 ft/sec
Duration of Operation	45 sec
Motor Weight	50 lbs
Chamber Temperature T_c	5000°F
Propellants	Acid-Aniline
Mixture Ratio r	2.75
Coolant	Aniline (Fuel)

Geometrical Specifications

Throat Diameter d_t	2.17 in
Exit Diameter d_e	4.85 in

Overall Length 22.5 in

The motor is fabricated of a type of stainless steel which has relatively high thermal conductivity, and is provided with an expansion joint to allow for the high operating temperatures of the inner shell, which is welded rigidly to the outer shell. The aniline coolant is conducted spirally around the nozzle and combustion chamber before entering the injector manifold (regenerative cooling). The motor interior is chrome plated for resistance to corrosion and erosion.

2. <u>Effect of Application on Design</u>

The requirements of each specific application which most strongly influence the design of the rocket motor intended to fit that application are as follows:

a) Magnitude of thrust

b) Duration of continuous operation

c) Operation expendable or repeatable

d) Operating altitude

e) Permissible motor weight

a) <u>Magnitude of Thrust</u>

Motors of less than 100 lbs thrust are subject to clogging of the minute injection orifices. As motors increase in size above 1000 lbs thrust the ratio of length to diameter decreases. Compare for instance Fig. 20 and Fig. 21. So long as a suitable

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Fig. 21

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cooled vessel can be built to withstand a pressure of about 500 lbs/sq in both in tension and compression there appears to be no fundamental upper limit to the size of a rocket motor. A motor delivering one million pounds thrust, for example, would have a throat diameter of about five feet. Recent studies indicate that in the larger motors, considerable reduction in relative combustion volume can be made without much loss in the combustion parameter c*. This results in a motor having a "morning-glory" shape consisting of a tubular section followed by slightly constricted throat and an expanding cone.

b) <u>Duration of Continuous Operation</u>

A rocket motor of the type shown in Fig. 20 usually reaches thermal equilibrium in about 30 seconds. If the motor is to be used for periods shorter than this it may be built with heavy uncooled walls, and depend upon their heat capacity to keep it at a safe operation temperature. The duration of a "cooled" motor is limited only by the amount of propellant available, since erosion of the nozzle throat - the vulnerable point at which highest heat transfer occurs - usually does not occur before exhaustion of the propellant. The limit of duration of a rocket-powered airplane, determined by the propellant weight which it can carry per pound of rocket thrust developed, is of the order of one hour or less. If there is no assisting lift from the air, as in the case of a wingless missile rising vertically, the operation period is evidently limited by the necessity that the weight of propellant be less than the thrust. It is interesting to note here that a vertical acceleration of 2 g must be maintained for ten minutes in order to escape the gravitational field of the earth.

Certain ranges of duration of motor operation are typical of specific applications. Illustrative of these is the following group:

Application	Duration
Artillery Rockets	0.1 - 1.0 sec
Missile Launching	0.5 - 5.0 sec
Aircraft Take-Off	10 - 45 sec
Missile Propulsion	30 - 300 sec
Aircraft Propulsion	5 - 60 min

c) Repeatable versus Expendable Operation

In expendable service it is often possible to tolerate a certain amount of erosion and build a much lighter motor than would be required for repeated operation, particularly if no human passenger is involved, in which case safety factors may be reduced. If the motor need not be started and stopped at will, conventional valves may be replaced by burst diaphragms, resulting in a weight saving and simplification.

d) Expected Operating Altitude

As was pointed out in Part I, Section D, for each external pressure there exists an optimum nozzle whose ratio of exit to throat area gives maximum thrust. Consequently a rigid nozzle can be designed correctly for only one altitude. Thus a motor for sea-level use might have an expansion ratio ϵ of 3.5 whereas if it were to be used at 40,000 feet it would have a ratio ϵ of 11.0. For vertical flight a mean altitude is chosen. In the case of motors which are to operate outside the atmosphere, the expansion ratio is made as large as is consistent with structural limitations.

e) Permissible Motor Weight

For aircraft propulsion, the motor weight may be only 1 or 2 per cent of the total, and rather heavy durable construction may be used. For missiles the motor may be 3 to 10 per cent of the total weight, and it is desirable to have the lightest possible construction. With present techniques, motors ranging from 20 to 100 lbs thrust per pound of motor weight have been built.

3. "Characteristic Length" L* and Combustion Time

The purpose of the combustion chamber is to provide sufficient volume to allow time for reasonably complete burning of the propellant before the products of combustion reach a cross-section near the throat of the exhaust nozzle. In contrast with the nozzle, it is not possible to predict the optimum dimensions of the combustion chamber. For convenience in manufacture, it is usually made in the form of a cylinder of volume V_c and length l_c.

It is found experimentally that the ratio of volume V_c to throat area f_t, V_c/f_t, which we shall define as the "characteristic length" L* must exceed a certain minimum value for adequate combustion of liquid propellants. This value may range from 25 to 600 in depending on the propellant and its method of injection.

There will now be derived a relation showing that the time of combustion t_c during which the reactants remain in the chamber is proportional to characteristic length L* provided the following simplifying assumptions are made:

a) The propellant components are completely mixed at the injector end of the cylindrical chamber; and sufficient, although incomplete, combustion takes place so that the fluid leaving the vicinity of the injector may be considered to be a gas.

b) The velocity and temperature of the fluid, after leaving the vicinity of the injector, will be assumed uniform throughout the combustion chamber even though we know further chemical reaction is taking place.

The combustion time t_c is

$$t_c = l_c/v_c \tag{80}$$

where v_c, the velocity of the reactants parallel to the axis of the cylindrical chamber, is given by the equation of continuity

$$v_c = m/\rho_c f_c = \frac{mRT_c}{f_c p_c} \tag{81}$$

From mass flow equation (39), $m = \Gamma' f_t p_c/a_c$. Evaluating m in equation (81) and substituting v_c in equation (80) above, we obtain, with the help of equation (52) which defines c*,

$$t_c = \frac{\ell_c f_c}{f_t} \cdot \frac{a_c}{\Gamma' RT_c} = \frac{\gamma L^*}{\Gamma' a_c} = \frac{\gamma L^*}{(\Gamma')^2 c^*} \tag{82}$$

Here we have assumed $l_c f_c$ to equal V_c, the effective chamber volume. We may thus see from equation (82) that the time which the reactants remain in the chamber is directly proportional to characteristic length L^* and inversely proportional to characteristic velocity c^*. For example, a typical 1000 lb thrust acid-aniline rocket will have $c^* = 4500$ ft/sec, $L^* = 3.33$ ft, and $\gamma = 1.25$, giving $t_c = 0.0017$ sec. As the size and thrust of the motor increase, the necessary L^* does not increase correspondingly, resulting in a change in proportions such that the chamber becomes a smaller fraction of the total volume and the nozzle a larger fraction. This is apparent if one compares the 1500 lb thrust motor, shown in Fig. 20 with the 56,000 lb (V-2) motor of Fig. 21.

4. Typical Design Calculation

Three sets of quantities which must be known accurately in order to design a regeneratively cooled liquid rocket motor are the chamber and nozzle dimensions, the hydraulic and mechanical parameters of the injector, and the hydraulic and thermal parameters of the cooling ducts. Consider first the procedure for finding the basic motor dimensions.

The quantities which may be selected more or less arbitrarily are:

Thrust F (lbs)

Combustion pressure p_c (lbs/sq in abs)

External Atmospheric pressure p_o (lbs/sq in abs)

Propellant chemicals

Ratio oxidizer to fuel flow rates r

Data concerning the performance parameters of the propellant (Cf Table III) must be collected empirically beforehand. From the given conditions, we wish to find values of:

Throat area f_t (sq in)

Exit area f_e (sq in)

Chamber volume V_c (cu in)

Propellant weight flow rate mg (lbs/sec)

One proceeds to do this in the following sequence:

1) Select a value of thrust F, combustion pressure p_c, external pressure p_o; determine the propellant combination and (if a bipropellant) the ratio r of oxidizer to fuel weight flow.

2) Using empirical data collected from static rocket motor tests with the chosen propellant at this p_c and r, in conjunction with thermochemical calculations, determine the ratio of specific heats γ, characteristic length L*, and characteristic velocity c*.

γ lies between 1.2 and 1.3 for the propellants described in Table III, while L* often lies between 50 and 100 inches. Typical curves of c* as a function of combustion pressure and mixture ratio are given in Fig. 22 and Fig. 23. These curves indicate that c* varies rather slowly with p_c and r.

Fig. 22 and Fig. 23

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3) Use the ratio of specific heat γ and the pressure ratio p_c/p_o to calculate the nozzle coefficient C_F and nozzle expansion ratio ϵ as in equations (45) and (50). Plots of C_F and ϵ are shown in Fig. 7. These theoretical values of C_F must be given a small correction for friction and divergence of the nozzle.

4) Use the basic relation (48) $F = C_F p_c f_t$ to compute throat area f_t, and the known ratio $\epsilon = f_e/f_t$ to compute exit area f_e.

5) Use the definition equation (52) of characteristic velocity c^* to calculate total mass flow rate m (i.e. $mc^* = p_c f_t$). The individual mass flow rates of oxidizer (m_o) and fuel (m_f) can be computed from the mixture ratio $r = m_o/m_f$.

6) Determine combustion volume V_c from the relation $V_c = L^* f_t$, using the empirically measured characteristic length L^*.

7) The ratio of combustion chamber cross-sectional area f_c to throat area f_t is determined by a knowledge of heat transfer and structural strength factors. Heat transfer is increased if f_c is small; stress is increased if f_c is large. A typical compromise value of f_c/f_t is 6. When this ratio is fixed, the chamber length l_c is also determined. Using as an example the 1500 lb thrust motor described in Section B, paragraph 1., one arrives by this process at the

following specifications, listed in the order in which they must logically be calculated.

<u>Specifications Given:</u>

Thrust at sea level, F	1500 lbs
Combustion pressure, p_c	300 lbs/sq in abs
Average external pressure, p_o	8.5 lbs/sq in
Propellant	Acid-aniline
Mixture ratio, r	2.75

<u>Derived Specifications:</u>

Ratio of specific heats, γ	1.25
Characteristic velocity, c*	4600 ft/sec
Characteristic length, L*	73.4 in
Corrected thrust coefficient, C_F	1.35
Expansion ratio, ϵ	5.0
Throat area, f_t	3.7 sq in
Exit area, f_e	18.5 sq in
Total weight flow rate, mg	7.8 lbs/sec
Oxidizer flow rate, $m_o g$	5.72 lbs/sec
Fuel flow rate, $m_f g$	2.08 lbs/sec
Combustion volume, V_c	272 cu in
Chamber area ratio, f_c/f_t	5.85
Chamber diameter, d_c	5.25 in
Chamber length, l_c	11 in

The design specifications concerned with the injector and the coolant ducts for this motor will be discussed below under appropriate headings.

5. Propellant Injection

The purpose of the "injector" is to introduce the propellant into the combustion chamber in such a manner that mixing or atomization takes place quickly and uniformly at the minimum possible expense of fuel pressure. If the propellant is a monopropellant, the injector orifices are designed to produce a fine spray, facilitating atomization and evaporation. If the propellant is a bipropellant, the orifices are designed to produce impinging high-velocity streams, facilitating mixing.

If the pressure drop across the injection orifices is too low, irregular combustion and even acoustic oscillations may result. On the other hand, too high a pressure drop adds to the weight of pressurizing tanks and is undesirable. A compromise which is satisfactory for acid-aniline is to have the "dynamic head" q of the impinging streams such that $q = \frac{1}{2}\rho v^2 \geq 8640$ lbs/sq ft. where ρ is liquid density and v is velocity in appropriate units. In the case of liquid in turbulent flow through the short tubular orifices of a typical injector, the pressure drop Δp is related to the dynamic head q by the relation.

$$\Delta p = Kq \qquad (83)$$

where K is a dimensionless orifice coefficient lying between 1.2 and 2.0, depending upon the shape of the orifices and degree of turbulence. The rate of heat transfer to the combustion chamber walls is very sensitive to the geometrical configuration of the entering propellant streams. Since cooling rocket motors regeneratively is a difficult problem, the orientation of the jet must be very carefully controlled. Variations of a few degrees in these jets may cause local variations in heat transfer of 50 per cent to 100 per cent, and may be responsible for the failure by melting of the chamber wall, since no great excess of cooling is possible.

The Impinging Jet Injector

Fig. 24 illustrates the construction of a typical acid-

Fig. 24

aniline injector with impinging jet type mixing. This is one of the simplest of many possible arrangements of orifices. The orifices are made removable, so that their hydraulic contour and orifice coefficients may be carefully controlled. Minimum heat transfer to the chamber is secured with this injector when the direction of the resultant momentum of the streams after impingement is almost parallel to the axis of the cylindrical combustion chamber. See Fig. 25 for a typical orientation of

Fig. 25.

the streams. The orientation of the resultant streams may also affect the performance (c*) of the motor indirectly by changing the entrance temperature of the fuel coolant. Fig. 26 shows the

Fig. 26.

appearance of water flowing through a small impinging jet injector. The specifications for the injector of Fig. 24, used in the 1500 lb thrust motor described in paragraphs 1 and 4 of this section are as follows:

No. of pairs of jets	8
Fuel orifice diameter	0.096 in
Oxidizer orifice diameter	0.1285 in
Nominal Δp - fuel jets	67 lbs/sq in
Nominal Δp - oxidizer jets	100 lbs/sq in
Angle of resultant momentum	$B + 5^{\circ}$ (outward)

In this case the jets are designed for unequal pressure drop because the hydraulic circuit of the fuel line included a pressure drop due to the motor coolant duct.

Other Types of Injector

An interesting bipropellant injector in which the fluids impinge in an annulus rather than at discrete points is shown in Fig. 27. This injector produces somewhat better mixing and

Fig. 27

combustion efficiency than the impinging jet type, by allowing a converging cone of fluid to intersect a diverging cone. It seems also to produce higher heat transfer to the motor walls.

Injectors whose primary function is atomization rather than mixing are built quite differently, as may be seen in Fig. 28, which shows an injector used with the monopropellant

Fig. 28.

nitromethane in a 200 lb-thrust motor. Here the liquid is whirled in a spray head, so that it is broken into fine droplets by centrifugal action upon emergence. The heat transfer from this type injector is considerably less sensitive to orientation than that from the bipropellant types.

The art of injector design is as yet largely empirical, since little is known about the internal processes in a liquid rocket motor. Direct measurements of velocity, temperature, density, and chemical composition are very difficult because of the high temperatures of the reactants.

6. Combustion Initiation

Starting liquid rocket motors presents a number of serious problems not encountered in conventional power plants. A "warm-up" period at rated thrust is not feasible since the high rate of propellant consumption makes each unused second of operation extremely costly in loss of impulse-weight ratio. Moreover, combustion must begin promptly (within a few tenths of a second) else the delayed ignition of a chamber filled with accumulated propellants may cause a destructive pressure surge or "hard start".

In the case of propellants which ignite spontaneously upon contact, such as acid and aniline, safe ignition is secured by limiting the initial flow to a fraction of the normal full flow until combustion is well established. These particular components react first at their liquid interfaces, releasing heat at a rate proportional to the surface area or degree of mixing. This heat is dissipated at a rate which depends upon the state of agitation of the liquids and the shape and temperature of the chamber into which they are injected. If the heat flow balance is unfavorable, the ignition may delay until excess propellant has accumulated, resulting in a "blow". It is essential that the hydraulic system be so adjusted that both propellant components arrive simultaneously, and that the mixture ratio during the initial transient flow not deviate too far from the stoichiometric value. An initial flow of 1/10 to 1/6 normal will usually establish sufficient combustion pressure in 2 or 3 seconds that the transition to full flow may be effected in 1 or 2 additional seconds. One successful starting technique has been to insert rupture discs in the hydraulic lines which burst when the propellant feed pressure has reached a specified small fraction of its final value. The delay in building up to full pressure then provides the necessary partial flow.

Combustion Initiation with Catalysts

Hydrogen peroxide may be promptly decomposed if there is injected with it about 3 per cent by weight of saturated calcium permanganate catalyst solution. Sometimes the permanganate is directed against a target to assist in atomizing it. If a fuel is being combined with peroxide, the catalyst flow may be stopped within a second or two and the reaction will continue. This has proven to be a very reliable method of ignition. A typical catalyst-peroxide-fuel injector is shown in Figure 29.

Fig. 29.

Peroxide decomposition may also be initiated by passing it through a bed of catalyst stones.

Combustion Initiation with Sparks and Flames

Non-spontaneous propellants, such as nitromethane and the various liquid oxygen combinations, must be ignited in the vapor phase by a spark or flame. Consequently the ignition device must be placed where it will not be flooded by the initial flow of propellant. See for example the spark plug location in Fig. 28. Spark plugs are convenient for small motors because repeated starts may be made, although they have the disadvantage that the electrodes burn off in relatively short time. For large motors such as the V-2, a pyrotechnic device in the motor interior is used to secure greater heat release, as otherwise there is risk of quenching the ignitor.

Nitromethane has the unfortunate property that a pressure surge or "hard start" resulting from delayed ignition (or thermal decomposition in coolant ducts) may initiate a detonation wave in the nitromethane which can be propagated along the fuel supply line and even into the supply tank. For this reason the spark plug circuits used with nitromethane are equipped with cut-offs which terminate the current after a fixed maximum safe delay interval measured from the instant fluid begins to enter the chamber. It is interesting to note that nitromethane is difficult to ignite unless gaseous oxygen is present, although oxygen may be discontinued as soon as the combustion begins. Detonation "traps" have been developed which stop the progress of the wave beyond a certain point in the supply line by shattering the line and dispersing its contents prior to the arrival of the wave.

7. Heat Transfer in Rocket Motors

a) Magnitudes of Thermal Quantities:

A rocket motor operates under more severe conditions of high temperature and of continuous rate of heat release than any other heat engine. For these reasons problems of heat transfer are among the most acute and important in motor design. Motors may be broadly classified into two categories depending on whether the heat transferred from the high velocity hot gases is absorbed by the heat capacity of the motor materials ("uncooled" type) or whether all or part of the propellant is used to absorb the heat ("cooled" type). The latter type can be further subdivided into the kind in which the coolant liquid absorbs heat as it circulates in ducts around the motor and is then injected into the combustion chamber ("regeneratively cooled" type"), and the kind in which a part of the coolant liquid is injected directly into the motor in such a way as to provide a coolant film on the inner wall surface ("film-cooled" type).

It is not at once evident, whether the propellant flow can absorb the heat transmitted to the walls, and still remain in the liquid phase. Measurements of the total heat transmitted show that it is often possible to absorb this heat without boiling the coolant. A marked scale effect exists for cooling. The rate of coolant flow increases linearly with motor thrust, while the motor area to be cooled increases less rapidly. This means that in certain critical cases a given propellant can be used to cool a large motor but not a small one.

A description of the numerical magnitudes of the thermal quantities involved in typical regeneratively cooled motors may be of value in providing orientation.

The heat of combustion of the RFNA-aniline propellant, for example, is approximately 1800 Btu per lb. At sea level, with $p_c = 300$ lb/in^2, less than 50 per cent of this heat is converted into kinetic energy of the jet, and almost the entire remaining portion remains in the jet in the form of thermal energy. In the motor, between 2 and 3 per cent of the heat of combustion passes through the chamber and nozzle walls into the coolant, which returns it again to the combustion chamber when regenerative cooling is utilized. This amount of heat transmission is brought about by a density of heat flow q of about 1.0 Btu per sq in per sec in the motor chamber and 2.5 to 3.0 Btu per sq in per sec in the nozzle throat, which is the position of maximum heat transmission. The hottest industrial furnaces have a rate of heat transfer only about one tenth of that encountered in the rocket motor. In Fig. 30 is shown the distribution of

Fig. 30

heat flow density q along the axis of a 200 lb-thrust motor made of an aluminum chamber and a copper nozzle block.

The heat transmitted to the motor can be reduced by a factor of two to four by inserting refractory liners. However, refractory materials so far developed have a limited lifetime in rocket motors.

For the RFNA-aniline propellant the temperature of the chamber gases T_G is about 4500°F, falling to 3000°F in the nozzle throat and 2000°F or less at the exit section of the exhaust nozzle. Most available metals melt well below these temperatures. The fact that uncooled motors of even short duration can be built is due to the existence of a large temperature drop across the boundary layer between the main gas stream and the motor walls. The equilibrium temperature T_{WG} of the inner wall surface (next to the hot gases) in regeneratively cooled motors ranges from 800°F to 1400°F for steel alloys. This temperature is determined largely by the wall thickness (0.1 to 0.2 in for steel alloys) and the thermal conductivity of the material. The temperature of the wall next to the coolant liquid T_{WL} is usually limited to a value below the boiling point of the liquid, which, for example, is 650°F for aniline at 500 lb/sq in. Another steep temperature drop occurs across the boundary layer between the main body of the coolant liquid and the cooled side of the motor wall. This "film drop" amounts to about 200°F. A typical value of the mean liquid temperature T_L after absorbing the transmitted heat is 300°F.

The relationship of these various temperatures is illustrated in Fig. 31, and the magnitudes of the various thermal quantities described above are listed in Table IV.

Fig. 31.

TABLE IV

TYPICAL REGENERATIVELY-COOLED ROCKET MOTOR THERMAL QUANTITIES

(RFNA-Aniline Propellant, Aniline Coolant)

Propellant Heat of Combustion	1800 Btu/lb
Per Cent of Heat Combustion as Jet Kinetic Energy . . .	40 Per Cent
Per Cent of Heat Combustion as Jet Enthalpy	60 Per Cent
Per Cent of Heat Combustion Transmitted through Walls .	3 Per Cent
Chamber Density of Heat Transfer, q_c	1.0 Btu/in^2-sec
Nozzle Density of Heat Transfer, q_n	2.5-3.0 Btu/in^2-sec
Temperature of Gases in Chamber, T_G	4500°F
Temperature of Gases in Nozzle Throat	3000°F
Temperature of Gases in Nozzle Exit	2000°F
Temperature of Motor Wall on Gas Side, T_{WG}	1000°F
Temperature of Motor Wall on Coolant Side, T_{WL}	500°F
Coolant Film Drop, $(T_{WL}-T_L)$	200°F
Mean Coolant Temperature, T_L	300°F
Coolant Boiling Temperature at 500 lb/in^2	650°F

The temperature of the motor wall for a given temperature of the chamber gases is controlled principally by the temperature of the coolant next to the wall. For this reason it is desirable to have the "film-drop" ($T_{WL}-T_L$) as low as possible. The thermal conductance h of this film, which is defined as the heat transfer density per unit temperature difference across it, increases with the velocity of the coolant, and the film drop may vary from thirty or forty to several hundred Fahrenheit degrees. Consequently it is desirable to have the coolant velocity as high as possible, consistent with an acceptable pressure drop across the coolant duct system. In present designs the coolant velocities vary between 15 and 50 ft/sec. The velocity is usually increased in the vicinity of the nozzle throat. An effort is made to limit the total pressure drop through the cooling ducts to a value of the order of 50 lb/sq in.

Since only 2 to 3 per cent of the heat liberated in the chamber is transmitted through the walls, and even this small percentage is absorbed by the coolant with but a scant margin of safety, any factor which affects the heat transfer is of great importance in the design of a motor. As has been discussed in division 5 of this section, the heat transfer is very sensitive to small variations in the flow pattern of the propellant inside the chamber. It has been found experimentally that slight deviations in dimension during fabrication of the injector are capable of affecting both the performance parameter c* and the heat flow q by factors of 20 to 100 per cent.

A promising means for controlling the heat flow to the wall of a rocket motor, first used by the Germans in the V-2, is the technique of introducing small quantities of a liquid at many points distributed uniformly over the interior surface. The liquid so introduced is spread over the wall in a thin film and eventually evaporated. The essential advantage of this method, termed "film cooling", is that the screening film of coolant fluid is permitted to vaporize, thus increasing its heat absorbing capacity many fold over that of a system in which the fluid must remain in the liquid phase. It has the further advantage that the heat does not have to be conducted through the motor walls, resulting in a great saving in strength and weight. In an ideal film-cooled motor the motor walls would require no external cooling and would never reach a temperature higher than the boiling point of the coolant.

A logical extension of the film-cooling process is to increase the number of cooling orifices indefinitely, or in other words, use porous walls. The coolant fluid then oozes in uniformly at all points of the surface. Such a technique is sometimes referred to as "sweat-cooling".

The coolant film may be produced either by part or all of the propellant or by some additional liquid which may be inert (for example water) or may contribute energy to the combustion process. A notably successful example of the use of film cooling is the German V-2 rocket motor, in which about

3 per cent of the total mass flow is diverted to the alcohol cooling film. About half of this takes part in combustion and hence is not wasted.

b) Mechanism of Heat Transfer

The hot gases transmit heat to the exposed motor walls principally by convection and radiation. Direct conduction may be neglected. Theoretical calculations[20] of the heat transfer indicate that up to 30 per cent of the absorbed heat in the chamber is due to radiation. In the nozzle where temperatures and dimensions are smaller, radiation is not important. This situation may alter if it becomes possible to use significantly higher temperatures in the combustion chamber.

[20] These calculations involve a number of doubtful assumptions about the emissivity of the hot gases and their velocity distribution in the combustion chamber.

The important quantity of heat transmitted by convection to the chamber wall is difficult to calculate accurately. The density of convective heat flux q_c through the hot gas boundary layer or "film" is porportional to the temperature difference across the film.

$$q_c = h_G \Delta T \qquad (84)$$

where h_G is called the gas film coefficient or thermal conductance and may for example be expressed in units of $Btu/in^2\text{-}sec\text{-}{}^\circ F$. $\Delta T = T_c - T_{WG}$, where T_c is the chamber gas temperature and T_{WG} the inner wall temperature. If it is assumed that the same conditions of convective heat transfer exist in the chamber as in a long straight pipe - a very doubtful assumption - then one may use an equation by Karman[21] based on the analogy between fluid friction and heat transfer to calculate h_G as follows:

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[21]Ref. Karman, Th. von, "The Analogy Between Fluid Friction and Heat Transfer", ASME Trans. 61, p 705-710, (1939)

$$h_G = q_c/\Delta T = C_H c_p \rho_c v_c \qquad (85)$$

where c_p = specific heat at constant pressure

ρ_c = mass density of combustion products

v_c = velocity of chamber gases

C_H = dimensionless heat transfer coefficient

Using the equation of continuity reduces equation (85) to the more practical form

$$h_G = C_H c_p (m/f_c) \qquad (86)$$

which indicates the manner in which heat transfer depends upon mass flow and chamber cross section. The coefficient C_H is a function of flow conditions, Reynolds number and surface roughness. It varies greatly from one section of the chamber to the next. A rough calculation using Ref. 21, gives a typical C_H = 0.0022, a figure which can be in error by as much as 100 per cent.

The heat flux density is determined primarily by the conductance h_G, since its value is much smaller than the other conductances in the heat flow path, in a manner analogous to the current in a series electrical circuit being determined by a predominately large resistor. A typical value of h_G is 0.0003 Btu/in^2-sec-$^\circ$F.

In practical calculations of rocket motor wall temperatures, it is necessary to have prior empirical knowledge of q, the heat flux density. If this number is at hand, one can proceed stepwise to calculate all the critical temperatures of Table IV in the following sequence.

1) With a knowledge of the chamber area A, coolant weight flow w_f, specific heat c_p, ambient temperature T_a and heat flux q everywhere over the surface, calculate the mean coolant temperature T_L

$$T_L = T_a + (1/w_f c_p)\int_0^A q\,dA \qquad (87)$$

2) At various critical sections of the motor, such as the nozzle throat and the region of maximum T_L, assume a coolant velocity and calculate the <u>liquid film conductance</u>[22] h_L, which then permits calculating the

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[22]The calculation of h_L, the liquid film thermal conductance, is made with the aid of a semi-empirical formula obtained by methods of dimensional analysis. A typical formula of this type is the following, quoted from p. 168 of "Heat Transmission" by W. H. McAdams (McGraw-Hill, N.Y., 1942)

$$(h_L D/k) = 0.023\,(vD\rho/\mu)^{0.8}\,(\mu c_p/k)^{0.4} \qquad (88)$$

where h_L = film conductance for circular tube

D = diameter of circular tube

k = thermal conductivity of liquid

ρ = mass density of liquid

v = velocity of liquid

μ = viscosity of liquid

c_p = specific heat of liquid

The bracketed terms are all dimensionless quantities. From left to right, they are known as Nusselt, Reynolds and Prandtl numbers, respectively. Certain quantities in this equation are not accurately known for rocket propellant liquids; for example thermal conductivity and viscosity. A correction to h_L must be made for the effect of bending the duct into circular arcs of rather small radius. This correction is not accurately known, but may be as much as 25 to 50 per cent.

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temperature T_{WL} of the chamber wall-liquid interface:

$$T_{WL} = T_L + q/h_L \tag{89}$$

If this temperature is too far above the local boiling point of the liquid at the estimated local pressure, it may be necessary to choose new values of velocity, area, or weight flow. A typical value of h_L is 0.005 Btu/in^2-sec-$^\circ$F.

3) Estimate the necessary thickness d of chamber or nozzle wall to withstand operating stresses at estimated operating temperatures, then knowing the

thermal conductivity K of the wall material, calculate the temperature T_{WG} of the gas-wall interface using the customary equation for heat flow through a slab. The nozzle throat will normally be a critical region.

$$T_{WG} = T_{WL} + qd/K \tag{90}$$

If T_{WG} is such that the material is too weak for the stresses involved, then a new choice of thickness, material, or T_{WL} must be made and the calculations repeated.

c) The Coolant Pressure Drop

Since feed pressure influences the critical parameter of overall weight, it is necessary to keep the pressure drop through the coolant ducts as low as possible. If the coolant velocities chosen in the preceding section cause too high a pressure, new velocities must be chosen and the calculations repeated. Care must be exercised in choosing the dimensions of the coolant ducts and in their subsequent fabrication, since at constant weight flow the pressure drop through a circular coolant tube, for example, varies inversely as the fifth power of the tube diameter!

Bending the cooling duct into a helix of moderate radius affects the flow in such a manner as to increase the pressure drop as much as 30 per cent. Using the best available hydraulic information, the pressure drop in the

1500-lb thrust WAC motor described in section 4 was computed to be 40 lbs/sq in, a figure which was later experimentally verified.

8. Characteristics of Rocket Jets

a) Shock Waves

A striking feature of the jet issuing from a rocket nozzle is the presence of clearly defined oblique shock waves[23],

[23] See Ref. 10, Vol. III. p 213-222, and also A Stodola, *Steam and Gas Turbines* (McGraw-Hill 1927) Vol. I, p 83-94 and Vol. II, p 1006-1016.

numbering up to six or more, such as can be seen in Fig. 32.

Fig. 32.

These consist of regions in which sudden, almost discontinuous, changes in pressure, density, velocity and entropy occur in the flowing gas. They remain fixed relative to the nozzle, and have no effect upon thrust unless they occur *inside* the nozzle, which will only be the case for a markedly over-expanded nozzle. It has been shown by Prandtl[24] that the spacing d of these

[24] Prandtl, L., "Stationary Waves in a Gas Stream" Phys. Zeit., 5, p 599, (1904)

luminous shock waves is simply related to the thrust F by the equation

$$d = \sqrt{1/3\ F} \qquad (91)$$

where d is units of inches and F is pounds. The <u>overall</u> length l of a typical rocket flame may be roughly estimated from the equation

$$\ell = \sqrt{F/10} \qquad (92)$$

where l is in feet and F is in pounds. In the case of an acid-aniline motor the after-burning or luminous flame may be virtually eliminated by the addition of 6 per cent of KNO_3 to the acid.

b) <u>Augmentation</u>

When a typical rocket jet is permitted to escape at atmospheric pressure, kinetic energy is wasted which might be more efficiently used. If this jet be permitted to entrain surrounding air, part of the energy can be used to impart a directed velocity to additional mass, and thus supply increased momentum or thrust. A tube, surrounding the rocket, in which this mixing may occur is called an "augmentor". A diagram of such a tube is shown in Fig. 33.

Fig. 33

Analysis indicates that up to 35 per cent thrust augmentation is possible with this type of tube if the system is stationary. Augmentation decreases rapidly if the system is in motion, falling to half the stationary value when the system is moving, relative to the atmosphere, with 5 per cent of the rocket exhaust velocity. In practice the necessary cylinders would be rather bulky to incorporate in an operating model.

Section C. Liquid Propellant Rocket Systems and Their Applications

1. Basic Components

A complete, self-contained liquid propellant rocket system must include not only the motor but flow control devices, a propellant supply in suitable containers, and a means of pressurizing the propellant. Figure 34 shows

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Fig. 34

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in diagrammatic form a typical gas pressurized system. A high-strength tank holds nitrogen at 2000 lbs/in^2. During operation, the supply pressure falls from 2000 lbs/in^2 to about 600 lbs/in^2. The nitrogen flows through a regulator which reduces the pressure to a constant value of 500 lbs/in^2. The regulated pressure is applied over both propellant components through a hydraulically or pneumatically operated valve and through one-way check valves which prevent possible disastrous mixing of the two liquid components via the nitrogen line.

The pressurized liquids are released to the rocket motor by a hydraulically operated valve (or in some cases by rupture diaphragms) placed as close to the motor as possible, thus facilitating the simultaneous arrival of the two fluids in the chamber. In a field service type of system, the propellant valve is arranged to open simultaneously with the nitrogen valve, which then permits liquid to flow during a gradual increase of nitrogen pressure in the ullage above the liquids and consequent smooth build-up of combustion pressure in the rocket motor. It is interesting to note in this connection that care must be taken not to permit the monopropellant nitromethane to flow too rapidly into the motor injector manifold, or adiabatic heating of the air-vapor mixture compressed in the manifold may cause an explosion.

The fabrication of the pressurized propellant and gas vessels demands the best technique possible, since they must weigh an absolute minimum and withstand high stresses with only a small factor of safety.

2. <u>Feed Pressure Techniques</u>

For short-duration systems, the use of a tank of pressurizing gas at ambient temperatures and at pressures between 2000 and 3000 lbs/in^2 is quite common. If it be assumed that no heat is transferred into the gas as it expands from an initial pressure p_o to a final pressure p_f in a supply tank of volume V_o, and that the gas passes through a regulator and fills propellant tanks of volume V_p at a regulated pressure p_r, then analysis shows that these quantities are related by the equation

$$V_o/V_p = \gamma\, p_r/(p_o - p_f) \tag{93}$$

where γ is the ratio of specific heats of the pressurizing gas. In actual practice enough heat is absorbed during the operating period (of the order of one minute) that γ must be replaced by a smaller "effective" γ' which is empirically determined. Thus for nitrogen $\gamma = 1.40$ and γ' may be as low as 1.25.

A very significant saving in weight may be achieved if the pressurizing gases can be generated as they are used from a chemical reaction. This results from the fact that the container for the chemicals will be very small and need withstand only feed pressure rather than many multiples of feed pressure. Furthermore, since the gases are usually hot when generated, they may be used at an absolute temperature several times the normal ambient temperature of "bottled" gases, thus reducing the density and mass of gas required by the same factor. Gases may be generated conveniently from rocket propellants themselves, either liquid or solid. A considerable body of technique remains to be developed however before this valuable weight saving method comes into practical use.

As the duration of operation of a rocket increases, the tank weight necessary to contain "bottled" pressurizing gas also increases. Above some critical duration of the order of 1 minute (which is less the larger the rocket thrust) pressurization can be accomplished with a smaller weight penalty by using turbine-driven centrifugal pumps than by using stored gases.

It is not possible here to go into the design details of these pumping plants. The gas turbine can be conveniently driven at 10,000 rpm or so by using two or three per cent of the rocket propellant in a special decomposition chamber as a gas generant. The detail technical problems of developing pumps to handle fluids such as nitric acid and liquid oxygen are numerous. Small pumps have been driven by turbines which project directly into the rocket exhaust and are rotated "windmill" fashion. It should be pointed out here that whereas chemical generation of pressurizing gases reduces the weight of the gas container it does not reduce the weight of the propellant container. With a pump feed system the propellant container becomes a light weight low pressure vessel.

3. <u>Assisted Take-Off and Superperformance of Airplanes</u>

The greatest demands upon an airplane power plant are made during take-off, at which time it must accelerate a maximum load to flying speed in a limited time and simultaneously overcome air or water drag. Extra thrust from a rocket motor is very valuable during this period. The following examples show typical improvements in performance when the thrust of an aircraft is increased 30 to 50 per cent by means of rockets.

1) The take-off distance with normal load may be decreased to about two-thirds normal value. This is of value on carriers and small airfields.

2) The load may be increased 20 per cent with normal take-off distance. This is of value on long flights where extra fuel is needed, or under emergency overload conditions.

3) The take-off distance at 10,000 feet altitude is reduced from nearly twice the sea-level distance without rockets,

to approximately the sea-level value. This is of value on high-altitude airfields.

4) The rate of climb at any altitude may be almost doubled. This is of value as a safety factor immediately after take-off, or in escaping pursuit, overtaking an enemy, and avoiding anti-aircraft fire.

5) The speed of level flight may be increased about 25 per cent. This is useful for the reasons mentioned in (4) above.

Fig. 35 shows a small airplane taking off with 150 lbs

Fig. 35

of jet assistance. The increase in angle of climb is quite obvious. Another application of JATO (jet assisted take-off) is in the take-off and landing of "supersonic" ultra high-speed airplanes, which may have such small wing surfaces as to be unable to take off or land without rocket assistance and braking action. The rockets used may be solid, (Fig. 16), or liquid, Fig. 36 , depending upon their duration, and may be droppable

Fig. 36

or permanently installed in the airplane. Both types have been developed commercially. A typical weight break-down of a liquid rocket JATO system designed to give 1300 lbs thrust for

60 seconds is as follows:

Propellants	400 lbs
Pressurizing Gas	25 lbs
Tanks	120 lbs
Plumbing and Structure	75 lbs
Rocket Motor	50 lbs
Total	670 lbs

4. Sole Propulsion of Aircraft

The rocket is well suited to the propulsion of aircraft at very high altitudes and at speeds above that of sound, in the region where propellers are not effective. The duration of operation is long enough that only pump-fed liquid rocket systems are feasible. This duration may range typically from 10 minutes to an hour.

Rocket propulsion will be especially useful for the "flying laboratory" type of airplane designed to study the aerodynamics of transonic, supersonic, and even hypersonic flight. In this connection rockets have already been sent aloft with small airfoils attached to the nose. Supersonic aerodynamic data is sent back from these by radio to ground recorders. This device is designated the "RAFT" or rocket airfoil tester.

One pump-fed system, designated the "Turborocket", is shown diagrammatically in Fig. 37. In this system the liquid

Fig. 37

propellant is pressurized by means of pumps driven by a gas turbine. The gas turbine may be operated by the products of combustion of the same propellants as are used in the rocket motor, or by decomposing a separate fluid such as hydrogen peroxide, as is the case in the German V-2. If rocket propellants are used, it is sometimes necessary to add a diluent to the reaction to reduce the gas temperatures to a value which will not damage the turbine blades.

A turborocket system was used by the Germans in their rocket fighter ME-163B. This power plant provided 3300 lbs thrust during the climbing period of 3 minutes and by means of a throttling system provided 300-400 lbs thrust during cruising. It achieved top speeds approaching sonic velocity and was limited to a total flying time of 10 to 20 minutes. At take-off, approximately 50 per cent of the weight of the plane was propellant, the combination used being hydrogen peroxide and "C-Stoff".

Another spectacular German rocket aircraft, known as the "Natter", was launched vertically from a pole, climbing to 40,000 feet in approximately one minute. It then released a simultaneous barrage of two dozen or so rockets at an approaching bomber formation. In the next few moments the nose of the aircraft was mechanically detached by the pilot and discarded, and a web-type parachute released from the tail. The resulting deceleration effected separation of the pilot from the cockpit, and he completed the landing via his own parachute, alone and unaccompanied by any of the original expendable aircraft. It would be a blase pilot indeed who would find such an assignment dull!

5. Missiles and Sounding Rockets

The Germans developed a great variety of liquid rocket missiles, ranging from an anti-aircraft projectile 4 inches in diameter and 60 inches long, called "taifun", to the massive V-2, weighing nearly 14 tons (Fig. 38). It is not feasible to

Fig. 38

attempt to describe them all here. The famous - or infamous - V-2 will suffice to illustrate the nature of these missiles. It is propelled by liquid oxygen and methanol-water, and is stabilized during the early stages of its flight by graphite vanes which project into the exhaust flame. The rocket motor, which is made of ordinary mild steel about one-quarter inch thick, has several rings of small holes through which a few per cent of the fuel component of the propellant is admitted to form a protective cooling film over the inner wall of the combustion chamber. The structure of the V-2 is shown in Fig. 21, and some of its specifications are tabulated in Table V. below.

TABLE V.

Typical V-2 Specifications

Maximum Range	200 miles (Approx)
Maximum Altitude (Vertical Trajectory) . . .	100 miles (Approx)
Maximum Velocity (End of Burning)	5000 ft/sec
Length Overall	46 ft
Body Diameter	4.5 ft
Total Weight (Including Propellant)	27,300 lbs
Empty Weight (Excluding Warhead)	5100 lbs
Warhead Weight	1700 lbs
Propellants	
Fuel	75% Ethyl Alcohol 25% Water
Oxidizer	Liquid Oxygen
Mixture Ratio (Oxygen to Fuel)	1.25
Rocket Motor	
Nominal Full Thrust (Sea Level	55,000 lbs
Duration of Thrust	80 secs
Throat Diameter.	16 in
Exit Diameter	29 in
Combustion Pressure	225 lbs/in^2 abs
Thrust Coefficient	1.30
Specific Impulse (Sea Level)	202 sec

The first American rocket to achieve altitudes comparable to the V-2 was developed at the Jet Propulsion Laboratory of the California Institute of Technology, and is named the WAC CORPORAL. The propulsion unit has been described in detail in Section B, Part III. This rocket was relatively small, carrying a payload of 25 lbs, and was designed primarily for sounding measurements in the upper atmosphere. It contains no jet vanes for stabilization, but depends upon the aerodynamic stability of its fins to keep it on a vertical course. It is launched from a 100-foot vertical tower with a solid propellant (ballistite) "booster" rocket originally developed for the Navy under the name of "Tiny Tim". This rocket was modified to deliver 50,000 lbs thrust for half a second. Such initial launching is necessary in order that the WAC CORPORAL achieve an aerodynamically stable velocity before leaving the launcher. Fig.39 shows a night picture taken of the launching process which shows cessation of the large booster flame just as the missile flame begins and subsequent separation of the flight paths of booster and missile. The structure of the WAC is

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Fig. 39 and Fig. 40

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shown in Fig. 40, and certain specifications[25] are tabulated in

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[25]A general description of the WAC CORPORAL and the activities of the Jet Propulsion Laboratory of Guggenheim Aeronautical Laboratory, California Institute of Technology, is given in the July 1946 issue

of "Engineering and Science" published by the Alumni Association Inc., California Institute of Technology, Pasadena, California.

Table VI. A view of the WAC launching tower at the White Sands, New Mexico, proving ground is shown in Fig. 41.

TABLE VI.

Typical WAC-CORPORAL Specifications

(Model A)

Maximum Vertical Altitude	43.5 miles
Overall Length	16 ft
Body Diameter	1.0 ft
Total Weight (Including Propellant)	691 lbs
Empty Weight (Less Payload)	272 lbs
Payload	25 lbs
Propellant	
Fuel	Aniline with 20% Furfural
Oxidizer	Red Fuming Nitric Acid
Mixture Ratio	2.75
Thrust (Sea level)	1500 lbs
Duration of Operation	45 sec

Motor data has been given in Part III, Section B.

The authors gratefully acknowledge their indebtedness for much of the information in Parts I, II and III of this paper to numerous members of the staff of the Jet Propulsion Laboratory, GALCIT. A large part of the information here presented has been abstracted from a text[26] prepared by this staff and used at the California Institute of Technology.

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[26] "Jet Propulsion" prepared for the Army Air Forces. This volume is in process of being rewritten in an unclassified version. Attention should also be called to survey publications by Dr. M. Zucrow of which the author became aware after completion of this paper. These are: The rocket power plant, SAE Jour. (trans.) 54, 375, (1946), Jet propulsion and rockets for assisted take-off, ASME (trans.) 68, 177-186, (1946) and a book incorporating these papers to be published soon, Principles of jet propulsion, John Wiley, N. Y. (1946).

The subject of the future development of rockets is an interesting one on which to speculate. It is evident that the science of rocketry is still in its infancy. In Part IV, to follow, some analysis will be made of two as yet undeveloped fields, escape from the earth and the application of nuclear energy to rocket propulsion. In the first sections it is demonstrated that in order to project a payload of significant magnitude away from the Earth, rockets will need to be built which are much larger than the V-2. In the second section, some of the fundamental problems in the use of nuclear energy will be examined quantitatively.

Legends for Figures, Part III

Fig. 18 Consecutive motion picture frames taken 1/64 second apart of the spontaneous ignition of red fuming nitric acid and aniline when brought in mutual contact.

Fig. 19 A 1500 lb thrust rocket motor chamber, showing cooling ducts.

Fig. 20 Section of 1500 lb thrust rocket motor showing from left to right propellant injector, combustion chamber, and exhaust nozzle.

Fig. 21 Exposed aft end of V-2 rocket, showing motor contour and elaborate piping installation.

Fig. 22 Variation of propellant characteristic velocity c^* with combustion pressure p_c for nitric acid and aniline.

Fig. 23 Variation of propellant characteristic velocity c^* with ratio r of oxidizer to fuel flow rate for nitric acid and aniline.

Fig. 24 Section of a typical bipropellant impinging stream injector for a 1500 lb thrust rocket.

Fig. 25 Diagram of the resultant momentum of the stream produced by two impinging injector streams.

Fig. 26 Water test of the orifice alignment of a 200 lb thrust injector.

Fig. 27 Section of a bipropellant injector producing two intersecting conical sheets of nitric acid and aniline.

Fig. 28 Section of a monopropellant injector designed to produce a finely atomized spray by centrifugal action. Note spark plug for ignition.

Fig. 29 Section of a bipropellant injector (peroxide-nitromethane) in which ignition is accomplished by a preliminary reaction between a permanganate catalyst and hydrogen peroxide.

Fig. 30 Plot of heat transfer density q as a function of position along the rocket motor axis.

Fig. 31 Plot of temperature levels across a section of rocket motor chamber. Note the steep gradients in the gas and liquid "films" next to the solid wall.

Fig. 32 The exhaust jet of a 200 lb thrust rocket motor, showing the luminosity striations due to oblique shock waves.

Fig. 33 An augmentor tube for enhancing jet thrust. This device increases the momentum charge which can be produced by the kinetic energy of the primary jet.

Fig. 34 Typical hydraulic circuit of a liquid rocket system using compressed gas for propellant pressurization.

Fig. 35 Light airplane taking off with the assistance of small solid propellant rockets. Note large angle of climb.

Fig. 36 Typical liquid propellant assisted take-off rocket power plant of approximately 1000 lbs thrust. This system, which is designed for permanent mounting in the engine nacelle of an aircraft, was built by the Aerojet Engineering Corporation.

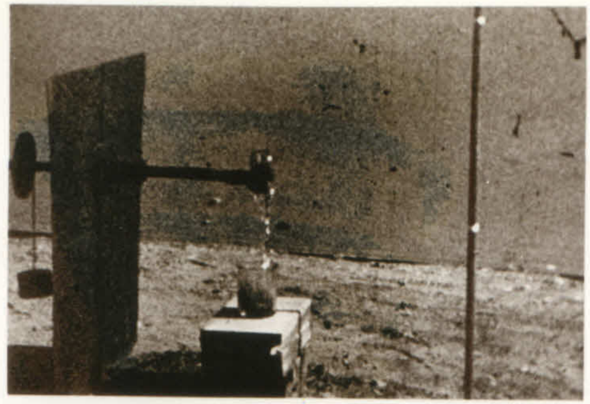
Fig. 37 Diagram of a rocket system in which the liquid propellants are pressurized by a turbine driven pump. (The "turborocket")

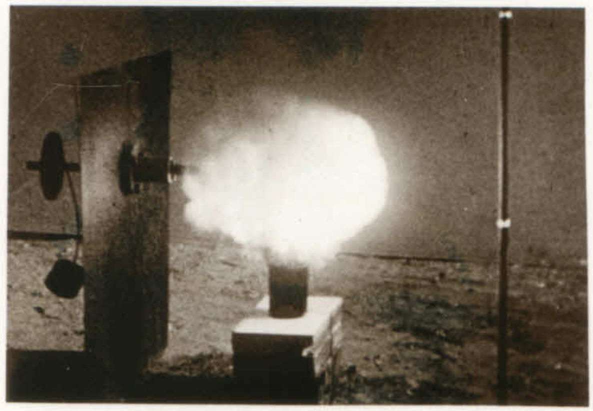
Fig. 38 A German V-2 rocket being serviced at White Sands Proving Ground, New Mexico, preparatory to firing.

Fig. 39 Night firing of liquid propellant sounding rocket with solid propellant booster, showing brief dark interval between termination of booster and initiation of missile combustion.

Fig. 40 The WAC CORPORAL sounding rocket which reached 43 miles altitude.

Fig. 41 Launching tower at White Sands Proving Ground, New Mexico, from which the WAC CORPORAL sounding rocket was launched. View of missile-proof observing room in background. Rockets have descended vertically from 40 miles to land close to this structure.





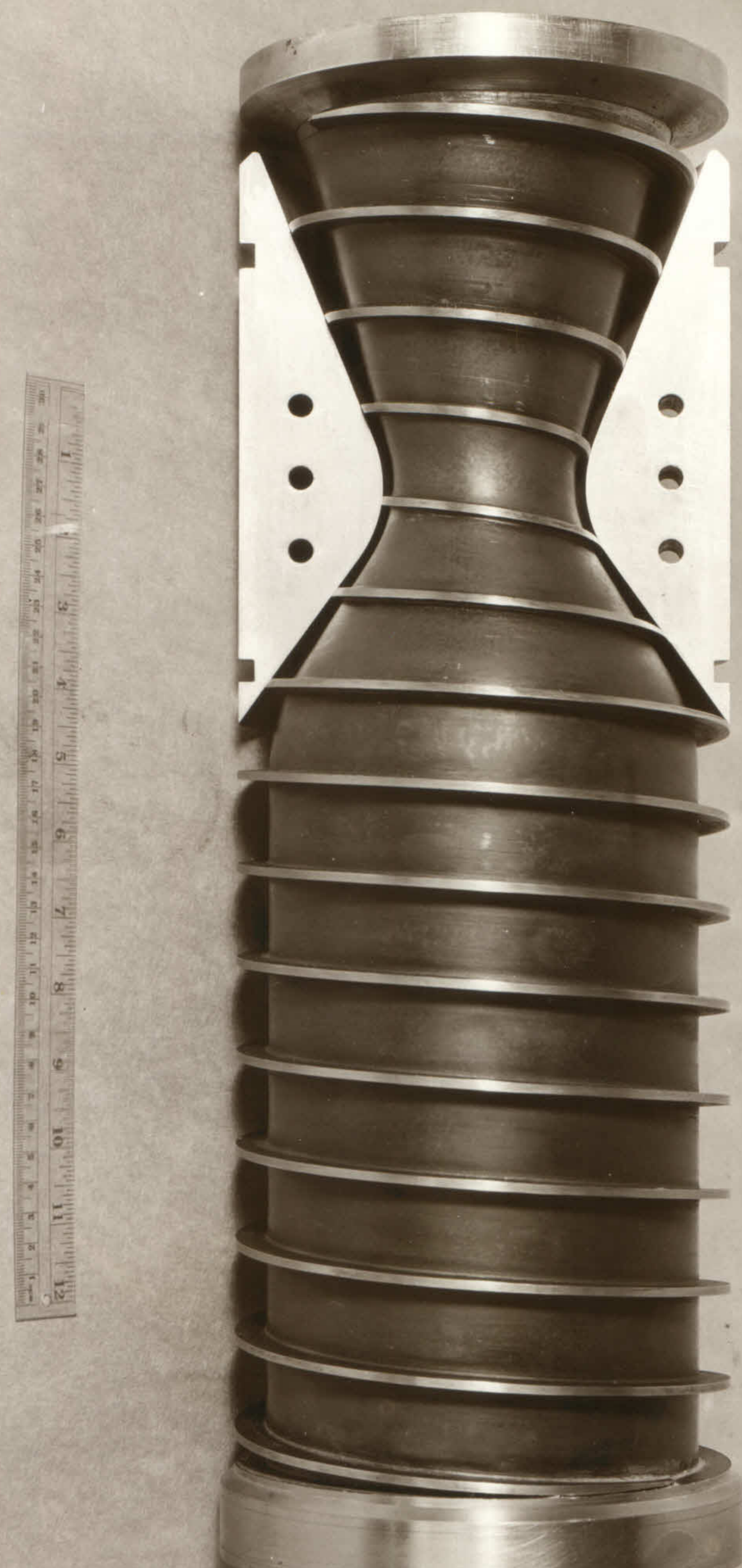
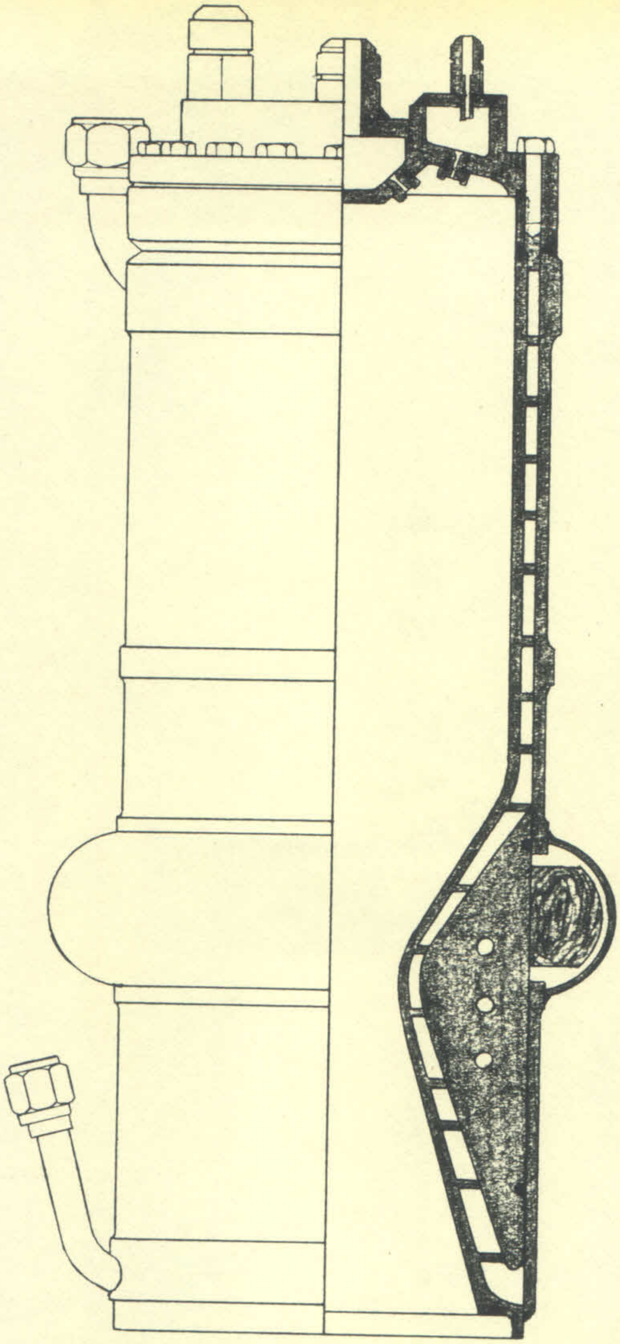
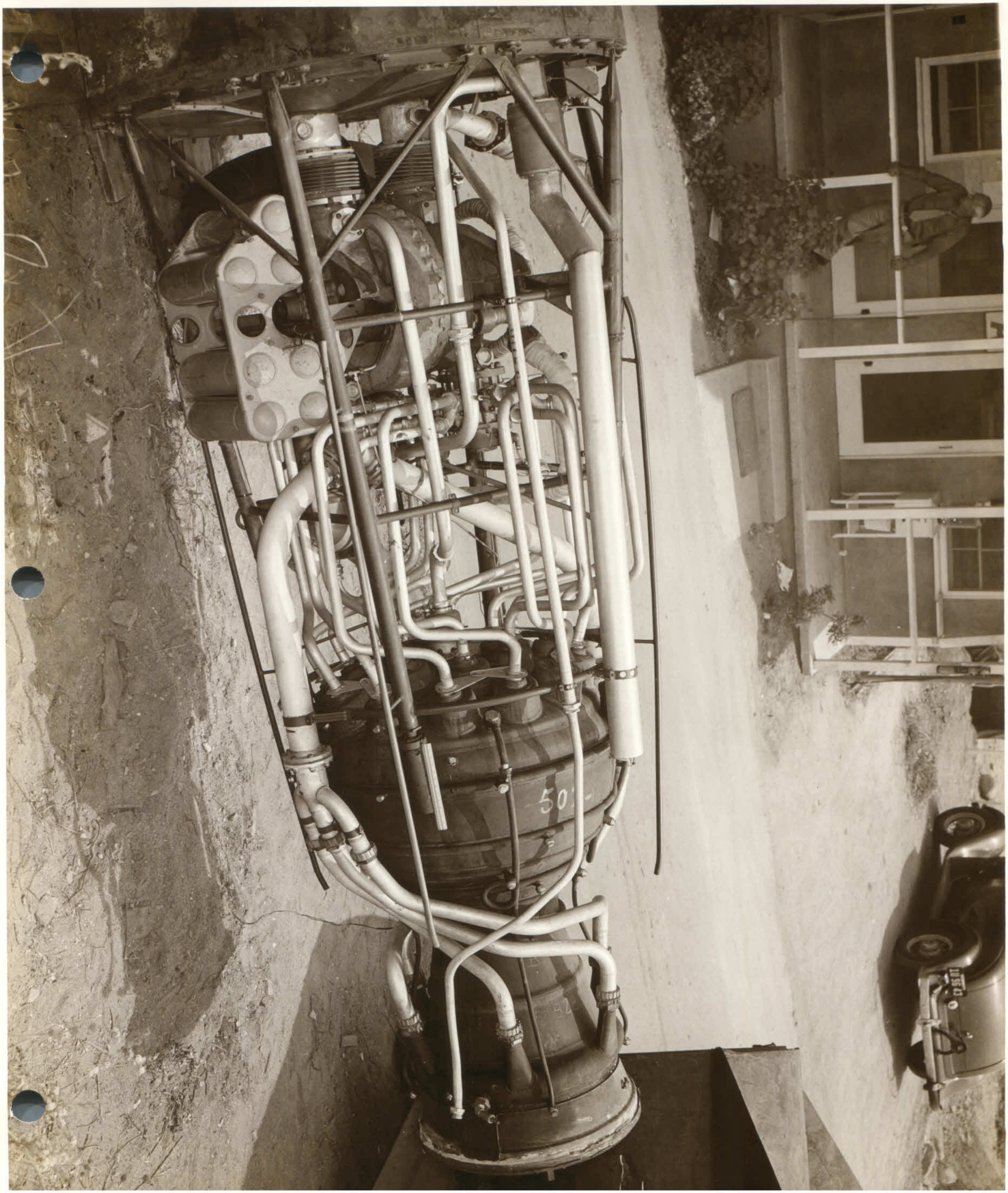


Fig. 20



RFNA - ANILINE
1500 LB WAC CORPORAL MOTOR
(ANILINE COOLED)

Fig. 20
H. S. Seifert
Physics of Rockets



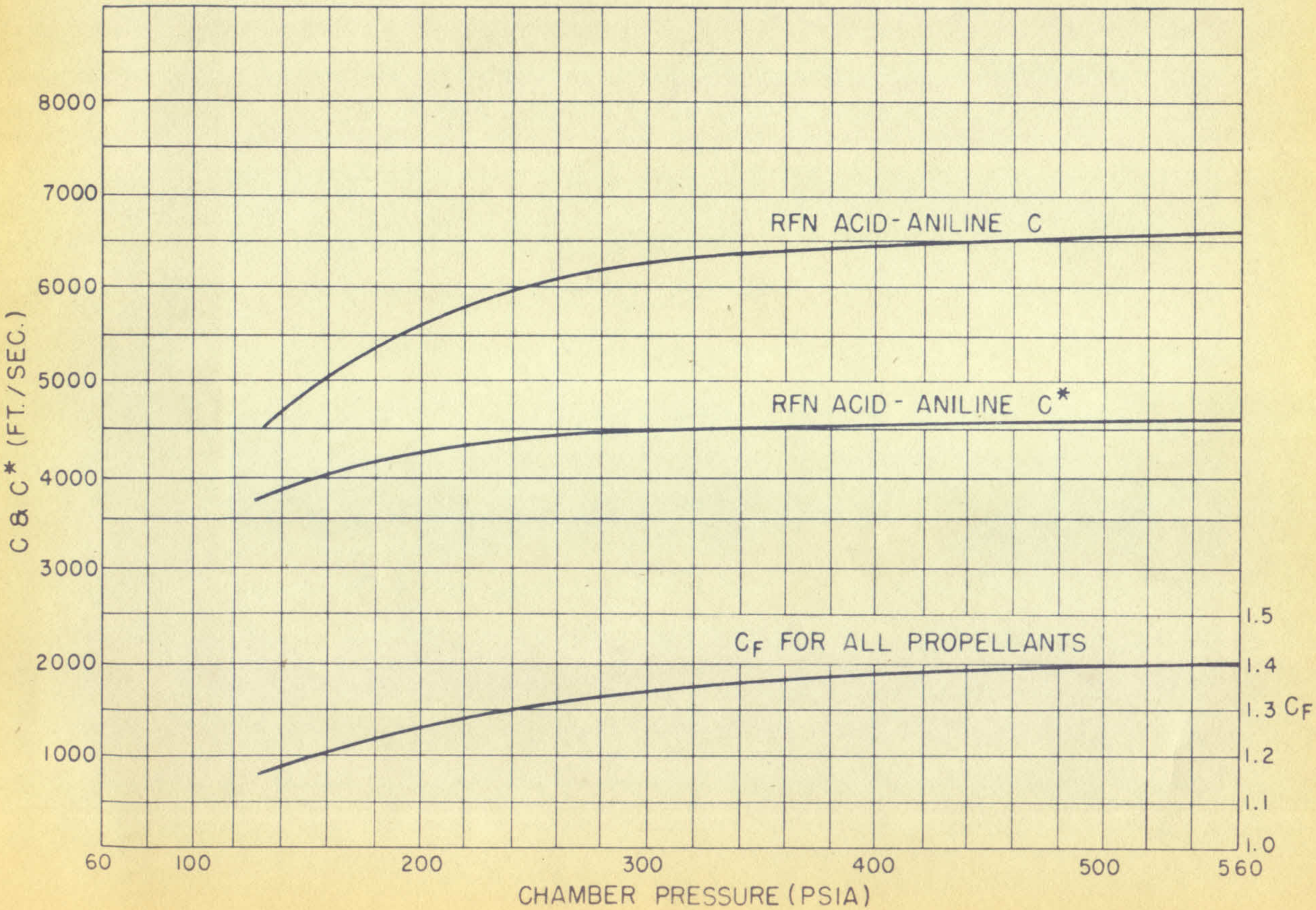


Fig. 22

Fig. 22
H. S. Seifert
Rockets

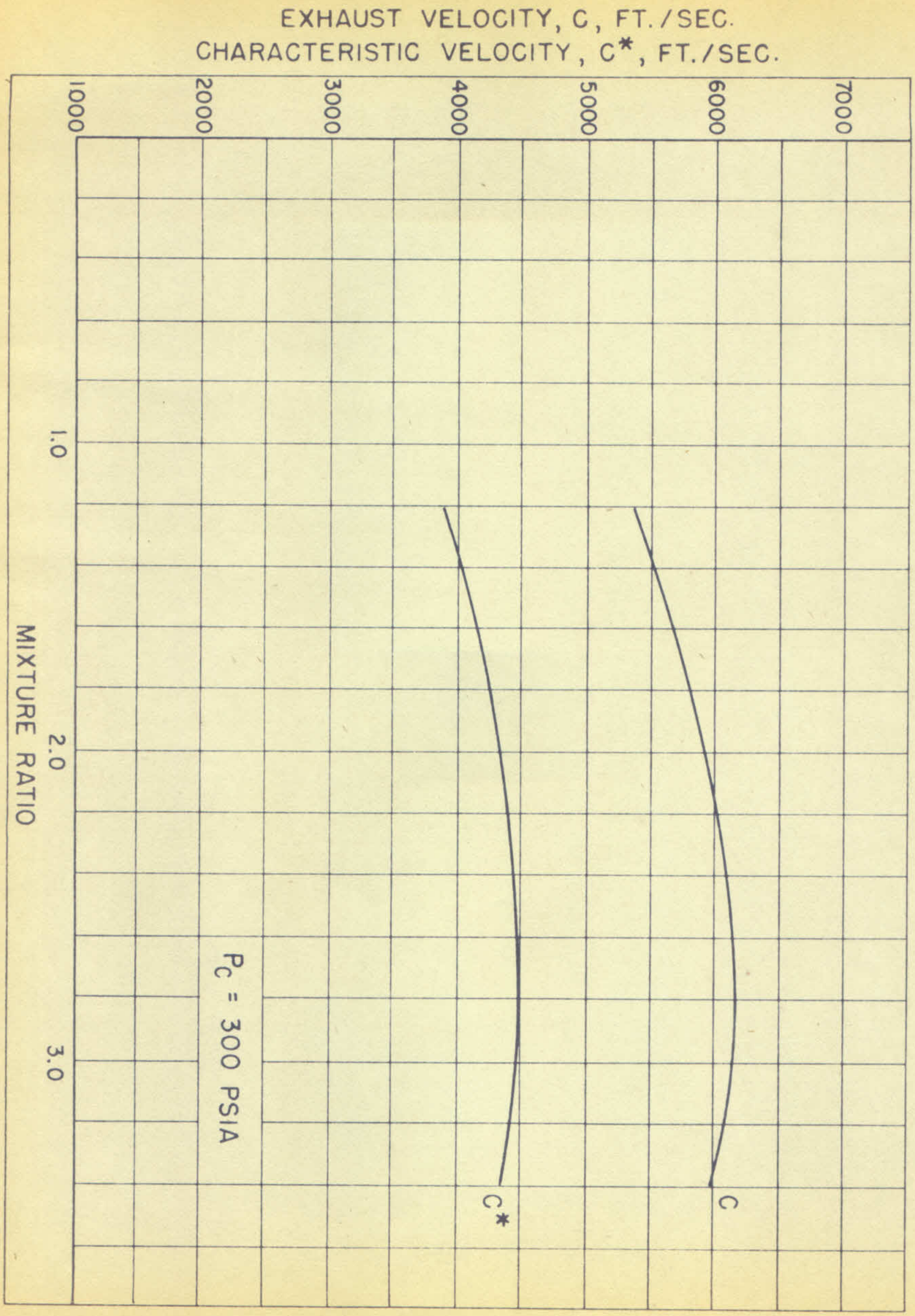


Fig. 23
H. S. Seifert
Physics of Rockets

Fig. 24

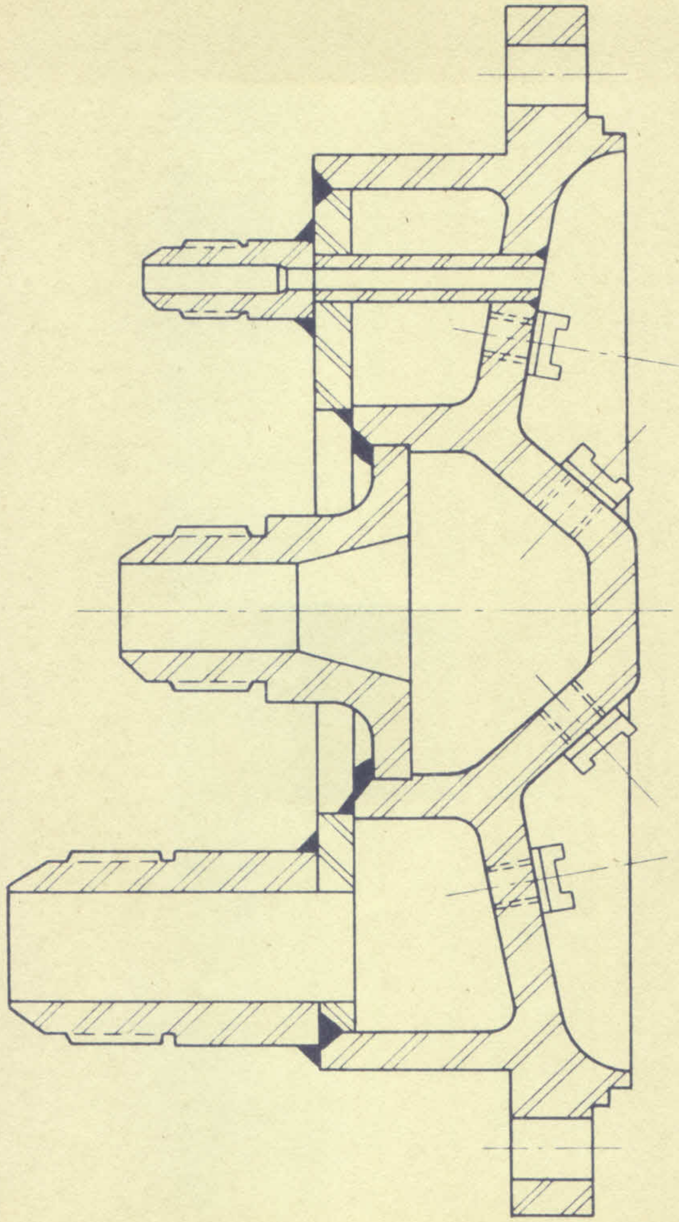


Fig. 24
H. S. Seifert
Physics of Rockets

Fig. 5

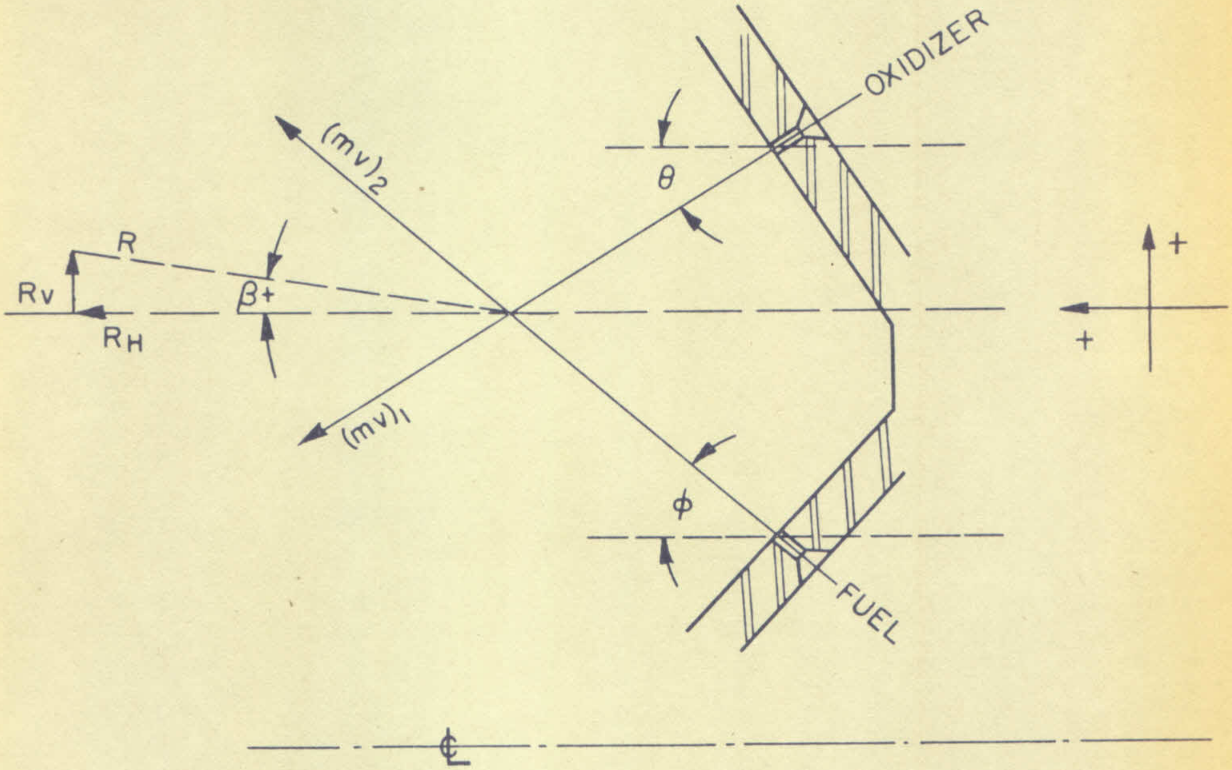


Fig. 25
H. S. Seifert
Physics of Rockets


2.72

Fig. 27
6

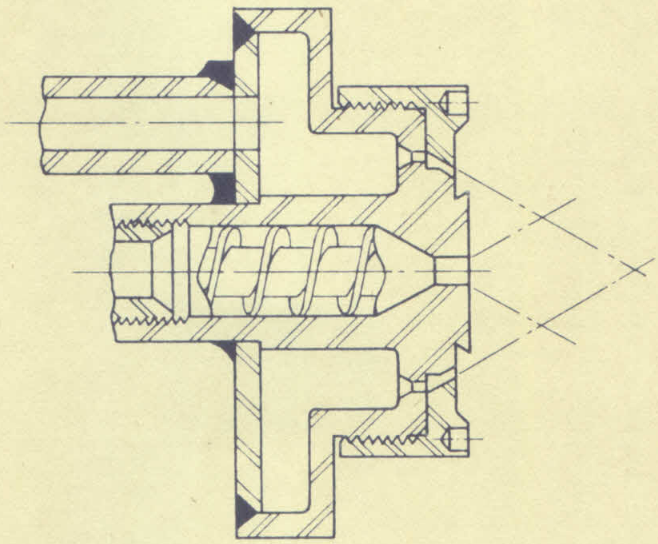


Fig. 27
H. S. Seifert
Physics of Rockets

Fig. 28

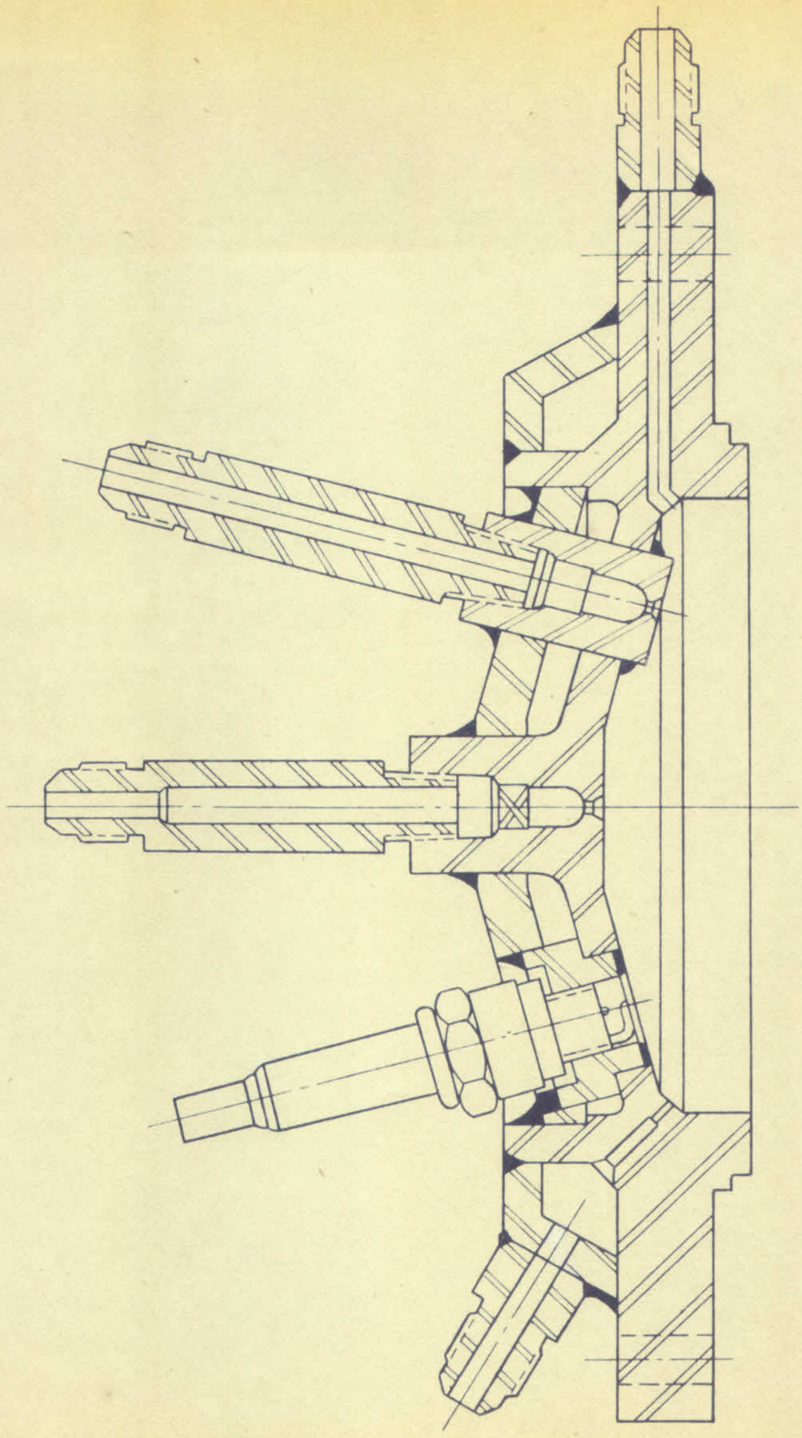


Fig. 28
H. S. Seifert
Physics of Rockets

Fig. 29

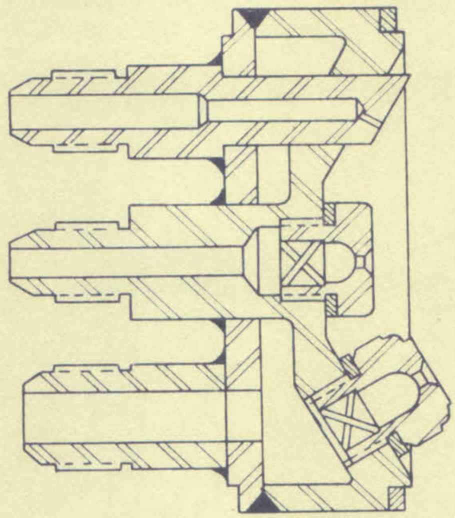


Fig. 29
H. S. Seifert
Physics of Rockets

Fig 30

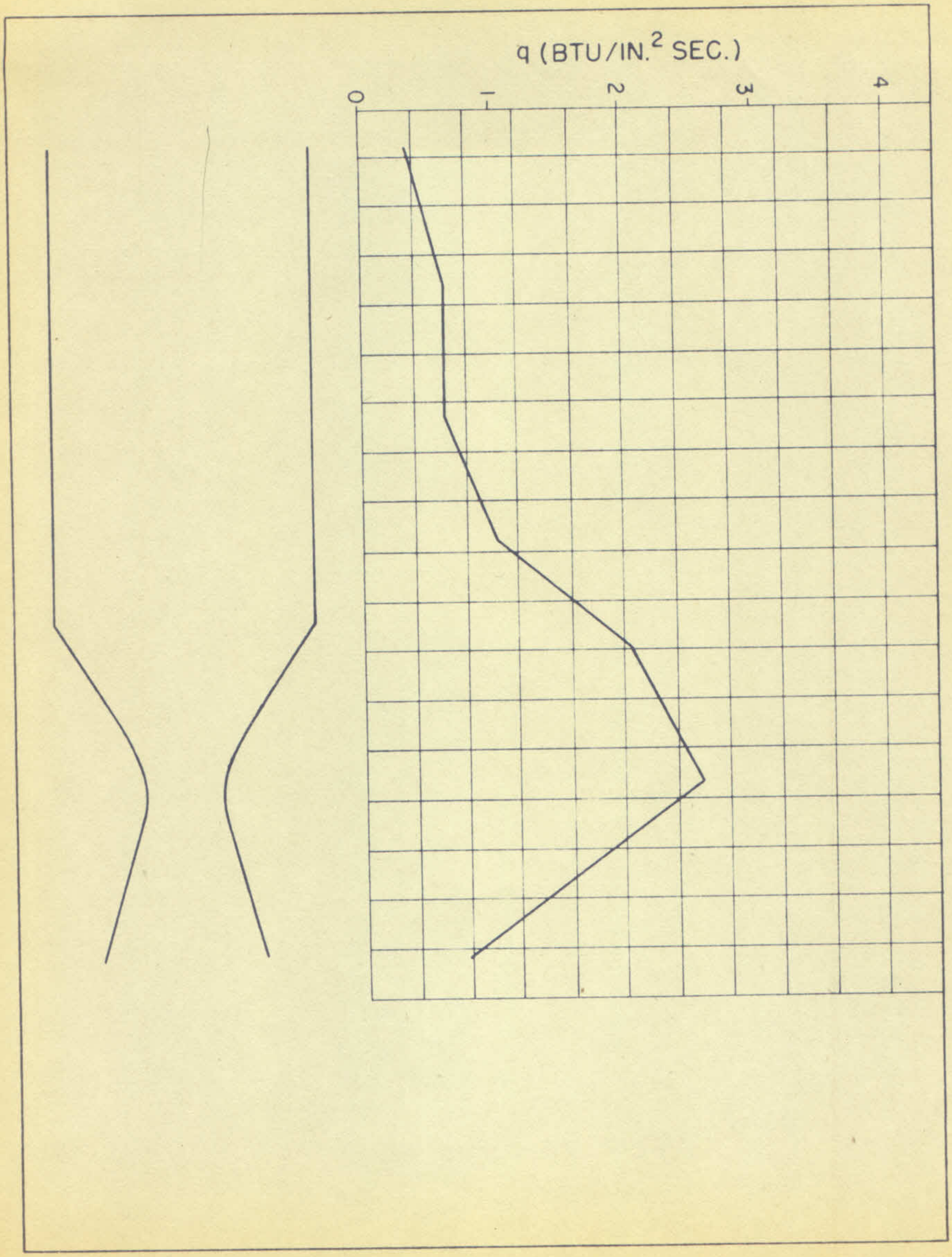


Fig. 30
H. S. Seifert
Physics of Rockets

Fig. 31

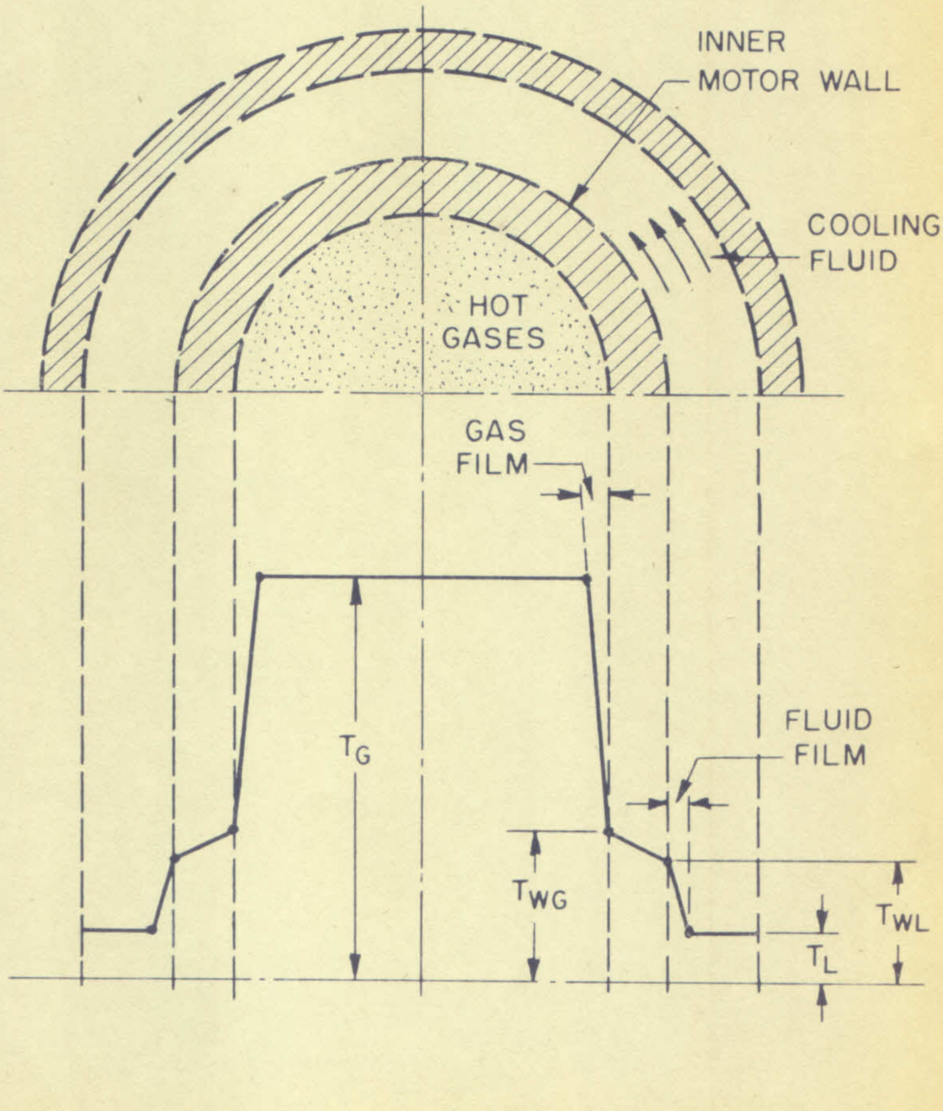


Fig. 31
H. S. Seifert
Physics of Rockets

Fig. 33

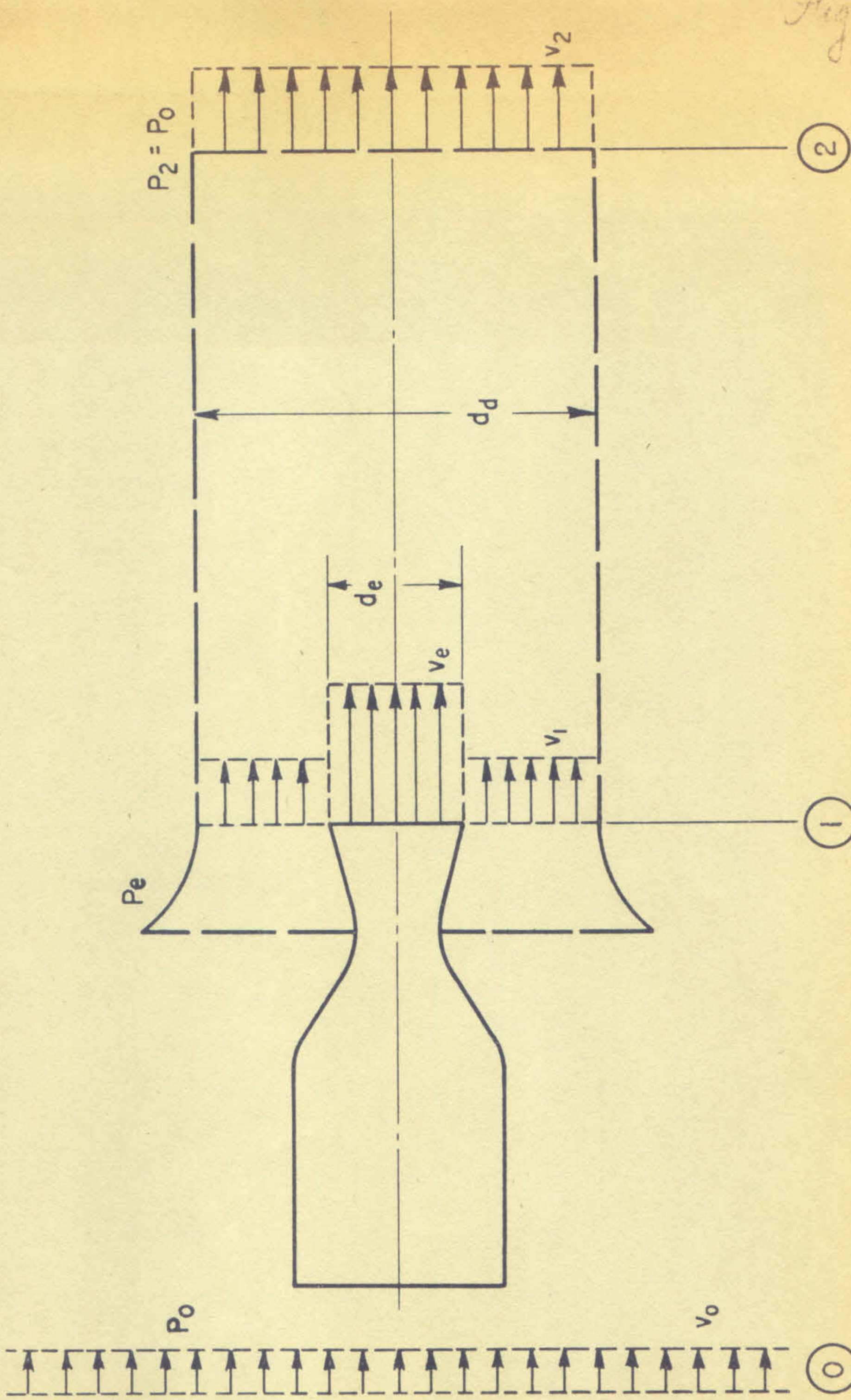


Fig. 33
H. S. Seifert
Physics of Rockets

Fig. 34

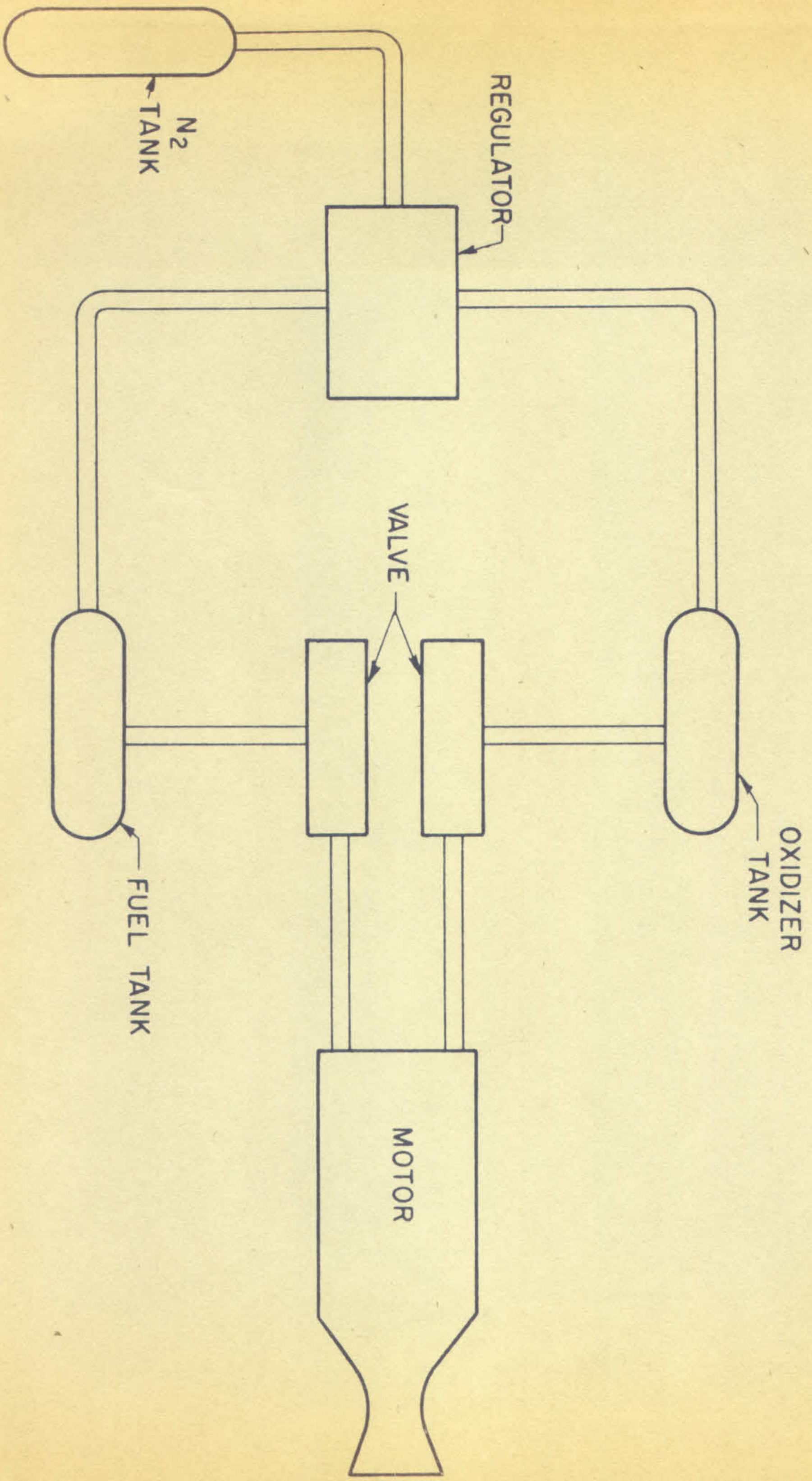


Fig. 34
H. S. Seifert
Physics of Rockets

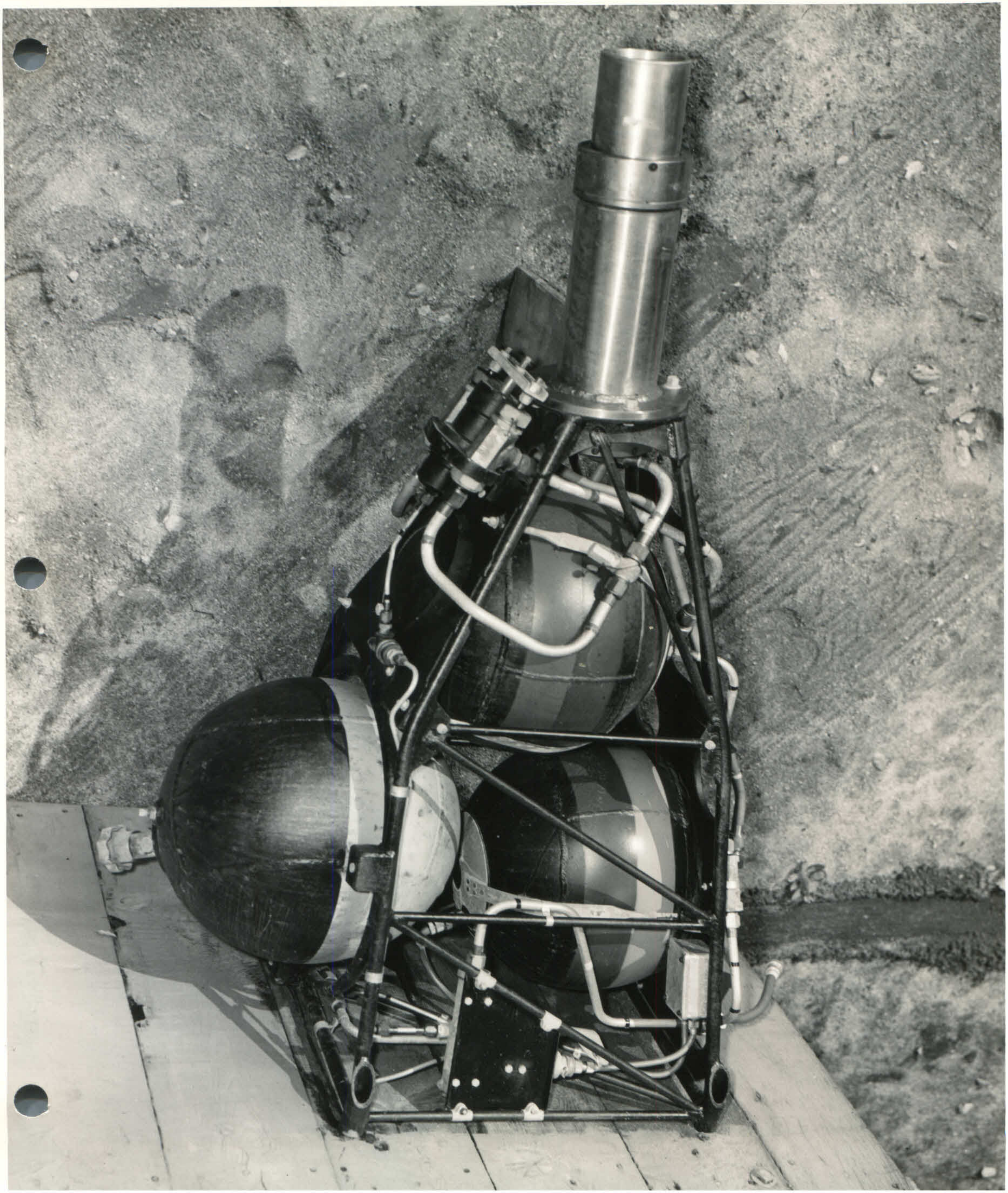


Fig. 37

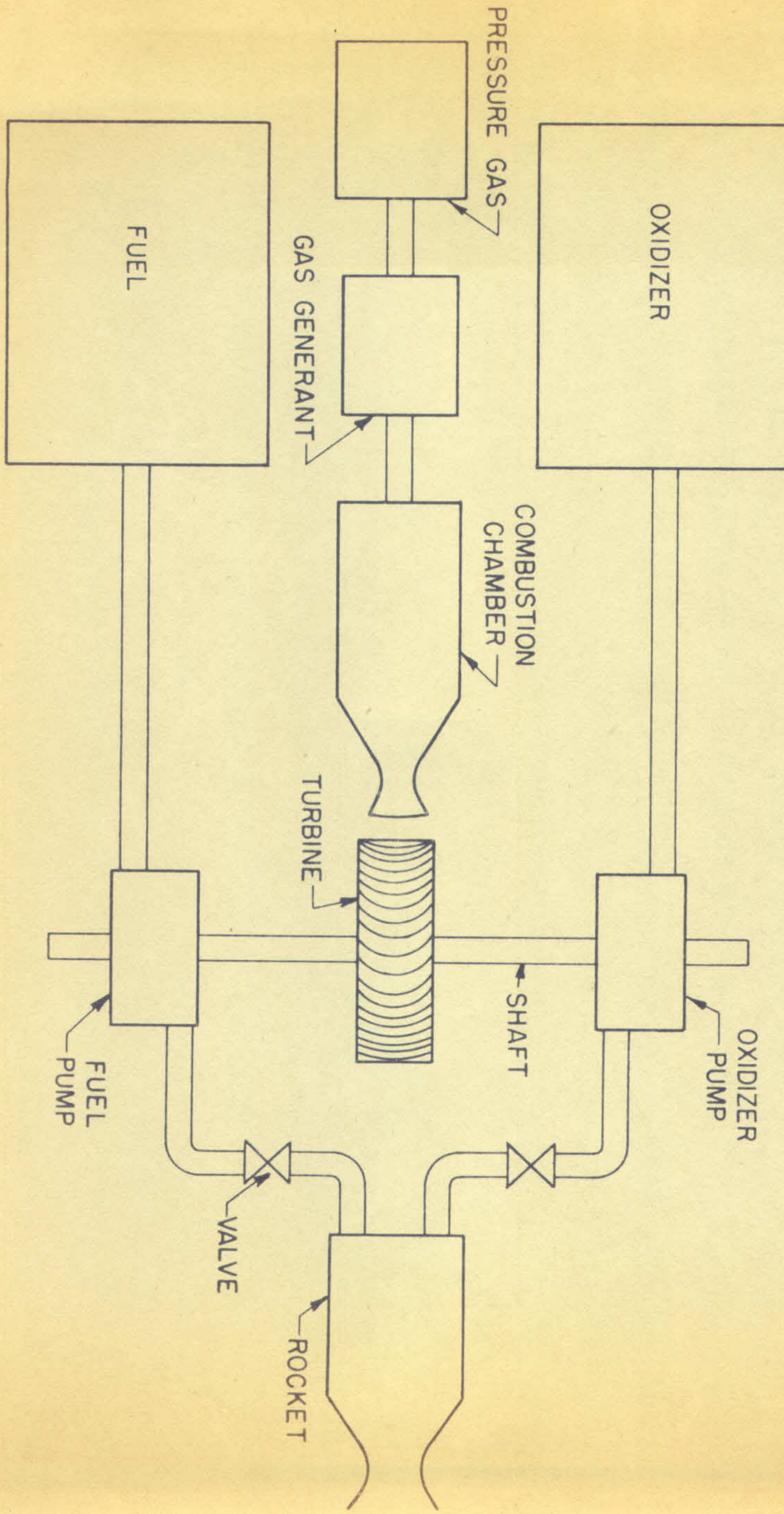


Fig. 37
H. S. Seifert
Physics of Rockets

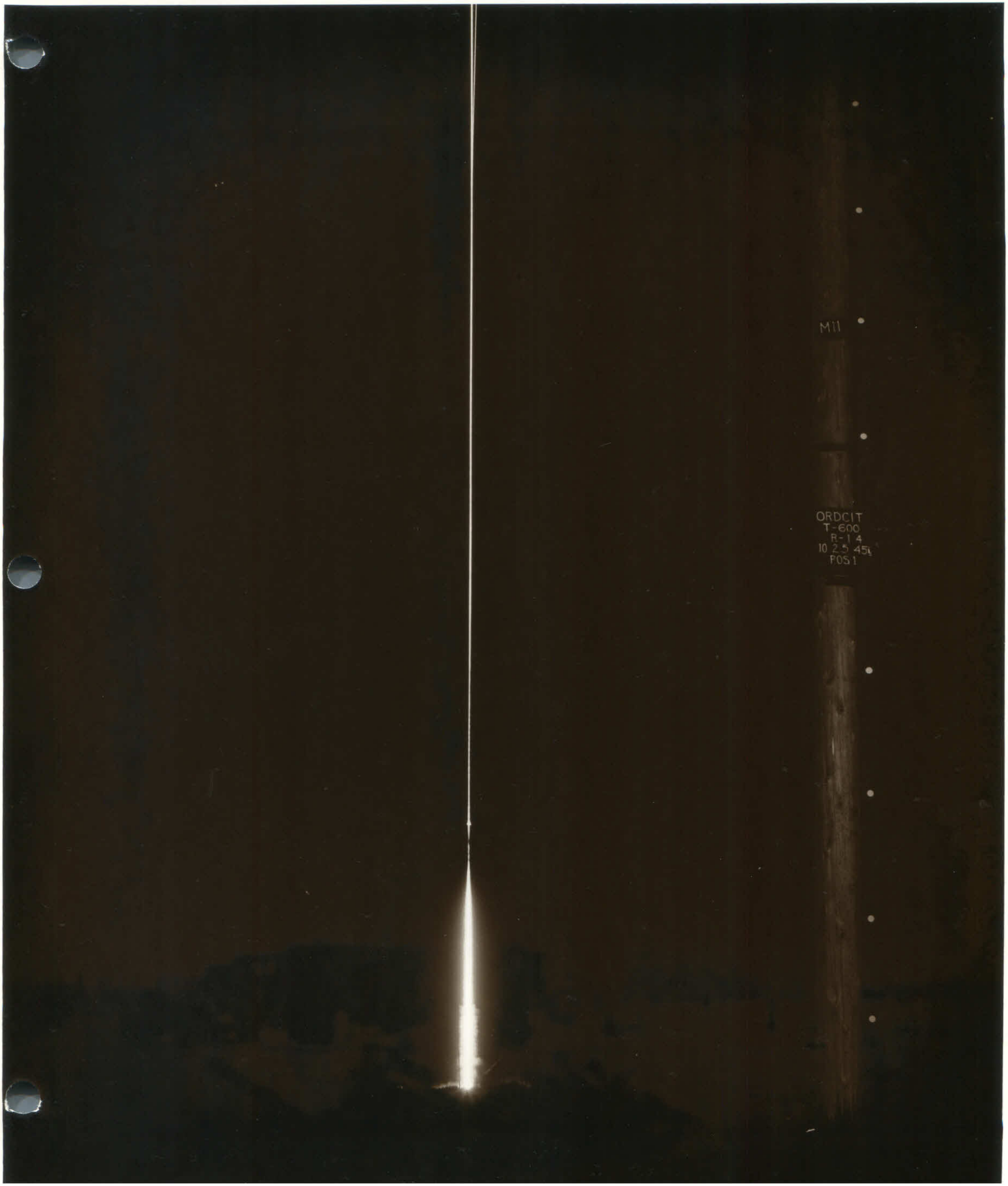

M11
ORDCIT
T-600
R-1 4
10 2.5 45
POS 1

Fig. 40

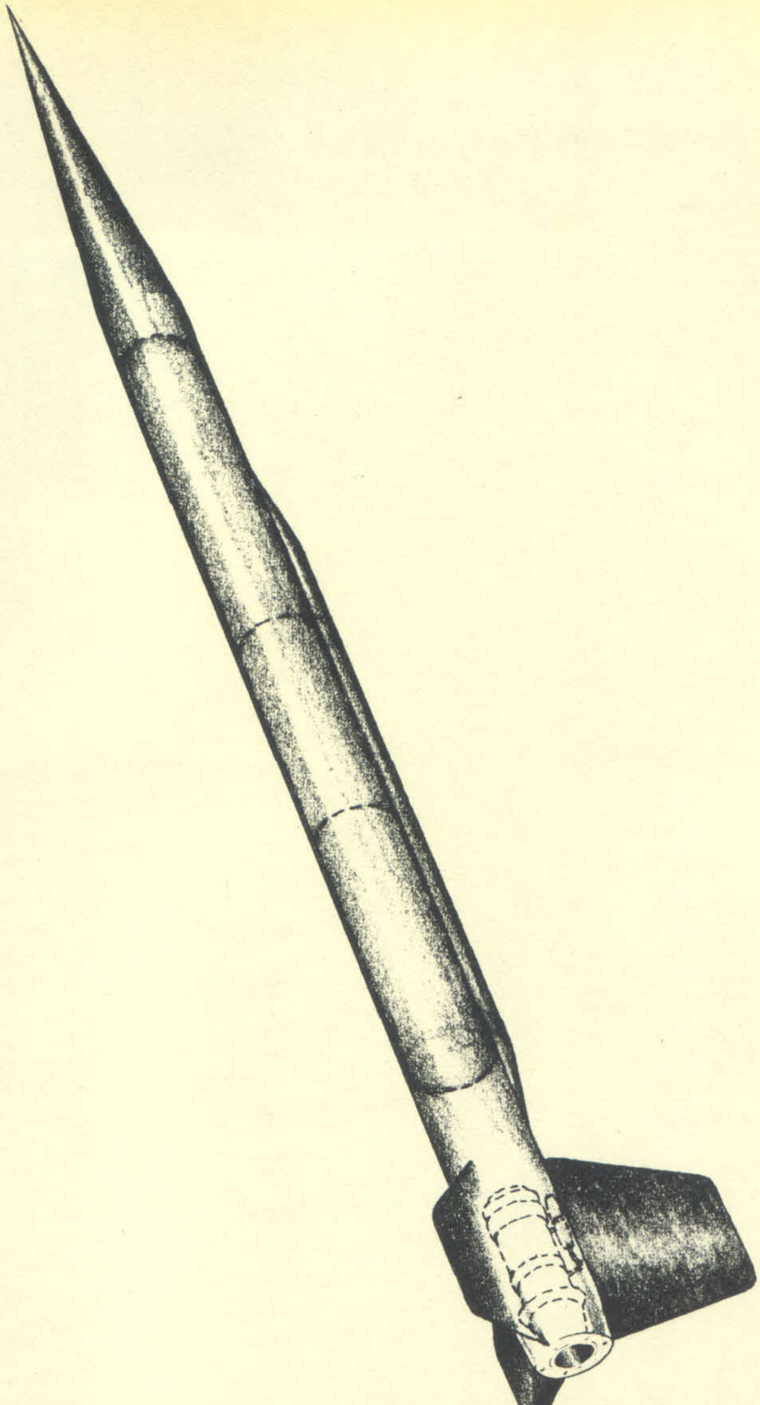
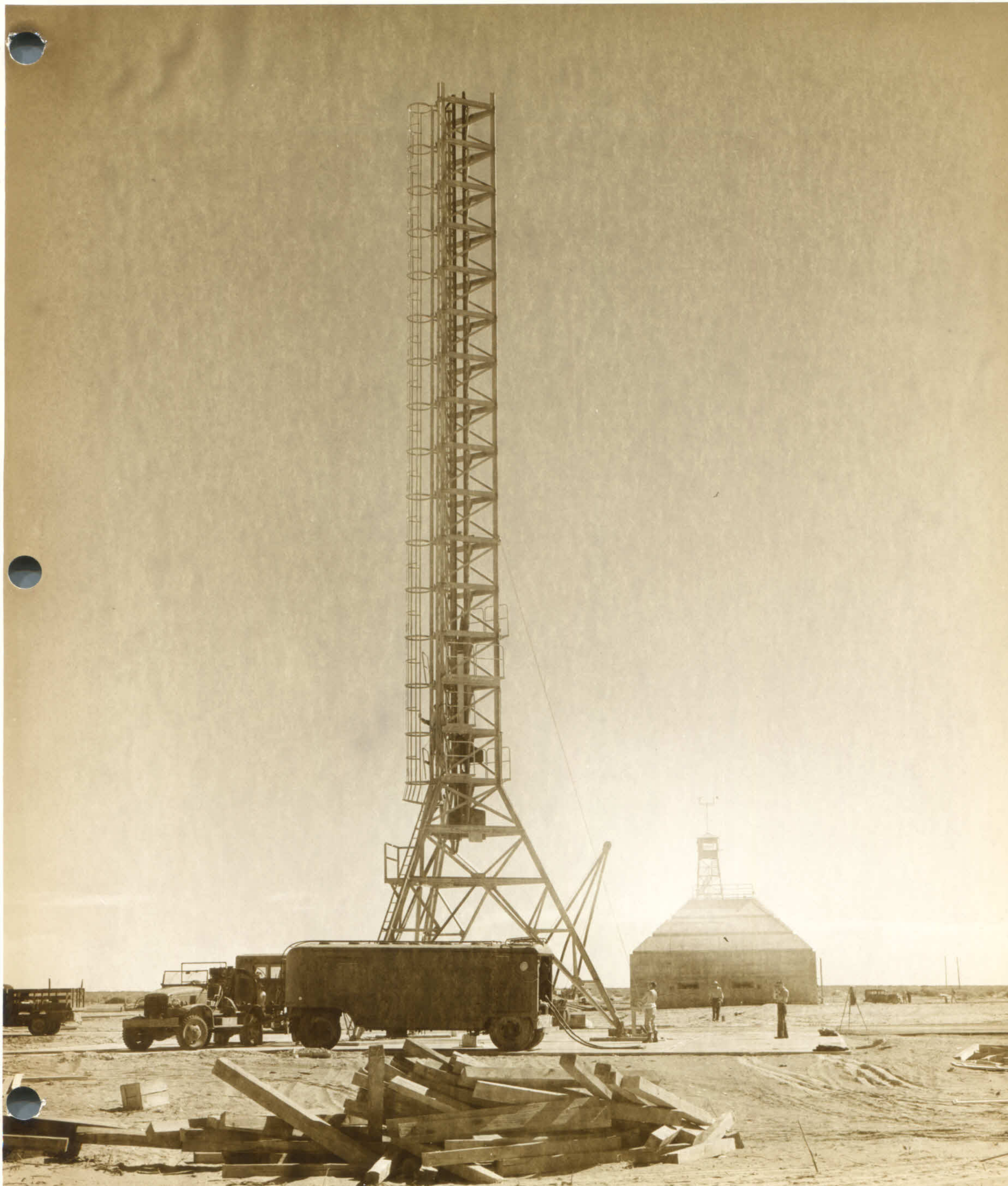


Fig. 40
H. S. Seifert
[illegible] Rocket



PART IV. DYNAMICS OF LONG-RANGE ROCKETS

Section A. Sounding Rockets and Escape from the Earth

1. History of High Altitude Rockets

The term "long-range rocket" is used to denote a vehicle whose range is at least 100 miles and which is propelled by a rocket whose burning time is, say, 30 seconds or more, the primary objective of the flight being the attainment of the longest possible range. These characteristics serve to distinguish this type of vehicle from the so-called artillery rocket, designed to carry a large payload over a short distance. A "sounding rocket" is a special type of long range rocket the purpose of which is to carry scientific instruments to extreme altitudes in a vertical trajectory, to determine the physical conditions prevailing in the upper reaches of the atmosphere and, indeed, beyond the earth's atmosphere. Since meteorological balloons have penetrated the atmosphere to a maximum altitude of only 25 miles, the useful region for the sounding rocket begins at that height.

The first technical discussion on record in which this problem is considered is that of K. E. Ziolkowsky published in Russian in 1903[27]. From his calculations he concluded

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[27] K. E. Ziolkowsky, "Rockets in cosmic space" (Science Survey, 1903).

that the potentially superior performance of liquid propellants as compared with powder-type propellants warranted research in that direction.

The earliest engineering attempts to achieve high altitudes by means of rockets were made by R. H. Goddard in the United States, starting about 1912 and continuing until about 1941[28].

[28]See Ref. 1, Part I of this paper, and also R. H. Goddard, "Liquid propellant rocket development", Smithsonian Miscellaneous Collections, 95, No. 3, (1936).

After determining the exhaust velocities obtainable from solid propellants, such as black powder and smokeless powder, and from the oxygen-gasoline combination, he designed and launched several liquid propellant rockets. The highest altitude of 7500 ft was reached in 1935 in the flight of a 14-foot rocket propelled by an oxygen-gasoline unit (Fig. 42). One of the interesting

Fig. 42

contributions of this work was the means of stabilizing the vehicle with gyroscopically controlled vanes acting intermittently in the jet blast.

In Germany, Hermann Oberth developed much of the theory of rocket performance during the period 1923 to 1930[29]. Active

[29]H. Oberth, Wege Zur Raumschiffahrt (Munich, 1929)

experimental work was conducted by the Verein fur Raumschiffahrt (Society for Space Travel), founded in 1927 to promote the development of the field of rocketry. Under its auspices, a number of liquid oxygen-gasoline rockets were developed and flight-tested. The highest altitude achieved in any of these flights was about 3000 feet, before this volunteer society disbanded in 1932, when the program was taken over by the German Ordnance Department[30].

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[30] W. Ley, Rockets , (Viking Press, 1941)

A similar volunteer group was organized in the United States in 1931 that now has the name of the American Rocket Society. Although its original objective was the development of high altitude sounding rockets, it had progressed only to the point of developing an oxygen-gasoline rocket motor before our entrance into World War II interrupted its activities[31].

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[31] G. E. Pendray, The Coming Age of Rocket Power, (Harpers, 1945)

At California Institute of Technology, interest in the possibility of achieving altitudes in excess of 100,000 feet was stimulated by the work of Malina and his associates, who began their investigations in 1936. Theoretical analyses of the flight performance of the sounding rocket and the thermodynamics of rocket motors were carried out during the period 1936 to 1940, and the characteristics of various propellants were investigated[32].

[32] See Ref. 2, Part I of this paper and also F. J. Malina and A. M. O. Smith, "Flight analyses of the sounding rocket", Jour. Aero. Sci. 5, 199 (1938); also H. S. Tsien and F. J. Malina, "Analysis of a sounding rocket propelled by successive impulses", Jour. Aero. Sci., 6, 50-58 (1938).

In 1940 the program at California Institute of Technology was expanded and directed toward rockets for war purposes. Interest in sounding rockets was actively revived in 1945, with the development of the so-called WAC CORPORAL sounding rocket, a vehicle designed specifically to carry about 25 lbs of instruments to high altitudes. Ten units were fired in August of that year, attaining a maximum altitude of 230,000 feet or 44 miles[33]

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[33] F. J. Malina, "Is the Sky the Limit?", Army Ordnance 31, No. 157, 45 (1946)

(Cf. Figs. ~~39~~ 40 and 43). The ~~fixing~~ firing of a solid propellant test vehicle used in preliminary development of the WAC CORPORAL is shown in Fig. 44.

Figs. 43 and 44

The highest altitude to date was reached by the German V-2 rocket missile when, in 1944, two units were fired vertically upward to a height of 107 miles[34]. In the normal ground-to-ground

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[34] A. V. DaRosa, "Analysis of V-2 Performance", Aero. Digest 98, May 1, 1945.

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trajectory the peak of the flight is about 60 miles high. In recent experimental launchings of captured V-2 rockets at the Army Ordnance Proving Ground, White Sands, New Mexico, altitudes ~~of about~~ above 100 miles have been achieved. This device represents the most advanced technique of rocket vehicle development at the present time (Cf. Figs. 45 and 46).

Figs. 45 and 46

In the next section the dynamics of the upward flight of a sounding rocket will be analyzed. By examining the influence of the various parameters that determine flight performance, the future possibilities of the sounding rocket can be estimated.

2. General Equations of Motion of a Rocket

A rocket propelled vehicle such as the V-2 or the WAC CORPORAL consists essentially of the following components: the propulsion system, a so-called payload that in a sounding rocket comprises the physical instruments being carried aloft, a quantity of liquid propellants, and the necessary shell and structure. The liquid propellants generally represent the largest portion of the gross mass of the vehicle. In the manner of the V-2, the rocket is set up on end, pointing vertically upward, for firing. If the rocket is not equipped with automatic guidance apparatus it must be projected from a guiding tower with sufficient velocity to be stabilized by aerodynamic forces.

In the following analysis only the flight performance of the rocket will be treated. Other aspects of the dynamical problem such as flight stability, the effects of control forces, and the stability of control mechanisms are too specialized for presentation in this paper.

During powered flight, the rocket is continually accelerated by the thrust delivered by the rocket motor which must be sufficient to overcome the retarding forces due to gravity and aerodynamic drag (Fig. 47). If the altitude at the end of powered flight

Fig. 47

is small compared with the radius of the earth, the variation of gravity may be neglected. However, the mass of the rocket, the aerodynamic drag, and the thrust delivered by the motor may vary during the flight of the rocket.

If F denotes the thrust of the rocket motor, D the aerodynamic drag, and M the instantaneous mass of the rocket, the acceleration of the rocket is given by the relation:

$$a = \frac{dv}{dt} = \frac{F - D - Mg}{M} \tag{94}$$

It is usual for the propellant flow controls of a rocket to be so designed that the mass of liquid ejected per unit time is constant. In this case the mass of the rocket decreases linearly with time, in the following manner:

$$M = M_0\left(1 - \zeta \frac{t}{t_p}\right) \text{ for } t \leq t_p \tag{95}$$

where M_0 is the initial mass of the rocket, t_p the duration of powered flight, and ζ the ratio of the initial mass of propellant contained in the rocket to the gross mass of the rocket.

The thrust at any time is expressed by the relation:

$$F \cancel{= \frac{M_p}{t_p}} = \frac{M_p}{t_p} c \tag{96}$$

where M_p is the initial mass of propellant, equal to ζM_0, and c is the effective exhaust velocity of the jet. The magnitude of c is a function of altitude, as explained above in Part I, Section 10, but to simplify the mathematics a constant average value of c may be used in (96) and in the remainder of the analysis. Therefore, although the thrust may be as much as 25 per cent greater at the end of burning than at the start, an average constant thrust will be assumed here.

The drag is determined as follows:

$$D = C_d A \rho \frac{v^2}{2} \tag{97}$$

where C_d is the drag coefficient based on the frontal area A, and ρ is the atmospheric density. The drag coefficient depends not only on the size, proportions, and surface qualities of a particular vehicle, but also on the velocity $\cancel{V}$ v. Its dependence on $\cancel{V}$ v is expressed in terms of its dependence on the Mach number, the ratio between flight velocity and the ~~prevailing~~ local velocity of sound. To simplify the considerations in this paper an average constant value of C_d will be assumed.

Combining equations 94, 95, 96 and 97, and integrating the resulting equation of motion, the velocity at any time t during powered flight can be obtained:

$$v = \int_0^t \frac{\frac{\zeta c}{t_p}\,dt}{1-\zeta\frac{t}{t_p}} - \int_0^t g\,dt - \int_0^t \frac{\frac{C_d}{\mu}\rho\frac{v^2}{2}\,dt}{1-\zeta\frac{t}{t_p}} + v_o \tag{98}$$

where v_o is initial or launching velocity. The parameter μ has been introduced to denote the ratio M_o/A, the magnitude of which determines the relative importance of the drag term.

The first two integrals can be integrated directly. The third integral may be calculated by numerical methods, and its value is denoted by $q_1\frac{C_d}{\mu}$, where

$$q_1 = \int_0^{t_p} \frac{\rho\frac{v^2}{2}\,dt}{1-\zeta\frac{t}{t_p}} \tag{99}$$

the velocity at the end of powered flight is then found to be:

$$v_p = -c\,\ln\,(1-\zeta) - gt_p - q_1\frac{C_d}{\mu} + v_o \tag{100}$$

In the case of drag-free flight, the velocity at the end of powered flight becomes:

$$v_{po} = -c\,\ln\,(1-\zeta) - gt_p + v_o \tag{101}$$

By integrating the differential equation twice, the altitude at the end of powered flight can be obtained:

$$h_p = ct_p\left[1 + \frac{(1-\zeta)}{\zeta}\ln\,(1-\zeta)\right] - \frac{gt_p^2}{2} + v_ot_p + ho - q_2\frac{C_d}{\mu} \tag{102}$$

where q_2 is the double integral with respect to t of the integrand of q_1. The effect of drag can be neglected by omitting the last term, $q_2 \frac{C_d}{\mu}$.

At the end of powered flight the vehicle continues to coast until its velocity is reduced to zero. The gain in altitude during coasting flight, if it is assumed to take place above the atmosphere, is given simply as follows:

$$h_c = \frac{1}{2g} v_p^2 \tag{103}$$

The altitude at the top of the trajectory is the sum of h_p and h_c, as follows:

$$\begin{aligned} h = {} & \frac{c^2}{2g}\left[\ln(1-\zeta)\right]^2 + c\,t_p\left[1 + \frac{1}{\zeta}\ln(1-\zeta)\right] \\ & + \frac{v_o^2}{2g} + h_o - \frac{v_o c}{g}\ln(1-\zeta) \\ & - \frac{C_d}{\mu}\left[q_2 + \frac{v_o q_1}{g} - \frac{c q_1}{g}\ln(1-\zeta) - q_1 t_p - \frac{q_1^2}{2g}\frac{C_d}{\mu}\right] \end{aligned} \tag{104}$$

The significance of each of the terms in equation (104) may now be examined. The first term, which is usually the predominant one, is proportional to the square of the exhaust velocity. Through its logarithmic factor, it is very sensitive to small changes in the propellant mass ratio ζ, especially ~~the~~ for values of ζ approaching unity. In fact, no matter how small the exhaust velocity, the altitude becomes infinite as the non-expendable mass of the rocket approaches zero. The second term is always negative, and since it is proportional to t_p, the longer the burning time, the lower the altitude of the peak of the flight. If t_p were the only variable, the maximum altitude would be attained if all the propellant were fired instantaneously. However, a rocket designed to withstand such large forces would require a much greater ratio of structural mass to propellant mass, with the result that the first term would be greatly reduced. Also, for flights starting from the earth's surface, a high velocity at the start of flight would increase the effect of drag. For these reasons, there is generally an optimum burning time that must be determined for each type of rocket. The third, fourth, and fifth terms are correction terms to be applied only when the rocket is started with an appreciable velocity before firing, as when booster rockets are used, or when the flight takes place from an elevated launching site.

The last term, negative in sign, contains the retardation due to drag. It vanishes, of course, for $C_d = 0$ or $\rho = 0$, that is, in vacuum or above the atmosphere. It is significant that the term becomes negligible for large values of μ. The parameter μ, defined as M_0/A, is proportional to the product of the length of a rocket by the average density. Thus, if similar rockets are compared, the effect of drag becomes negligible as the size of the rocket increases. Therefore, the performance of large rockets can be closely approximated by the first five terms alone, which contain no factors depending on size.

The altitude reached by a rocket large enough to cause the last term to vanish, when launched from a stationary position at $h_o = 0$, is given simply by the following:

$$h_o = \frac{c^2}{2g}\left[\ln(1-\zeta)\right]^2 + ct_p\left[1 + \frac{1}{\zeta}\ln(1-\zeta)\right] \quad (105)$$

The relative effects of c, the exhaust velocity, ζ, the propellant mass ratio, t_p, the firing time, and μ, the gross mass per unit frontal area, are illustrated in the graphs of Figs. 48 and 49, representing altitudes calculated by means of equation (104). These calculations were carried out with the aid of the drag curve given in Fig. 50. This curve is

Figs. 48, 49, and 50

considered representative of well-designed rocket vehicles. The atmospheric density and temperature distributions of the National Advisory Committee for Aeronautics (NACA) standard atmosphere were used[35].

[35] W. G. Brombacher, "Altitude-Pressure tables based on the United States standard atmosphere", Report No. 538, National Advisory Committee for Aeronautics, (1935).

The characteristics of both the WAC CORPORAL and the V-2 Rocket are presented in Table ~~VII~~ IX for comparison of actual performance with that predicted by means of equations (104) and (105), using the drag characteristics depicted in Fig. 50.

TABLE ~~VII~~ IX

Comparative Flight Data for WAC and V-2 Rockets

	WAC CORPORAL	V-2 Rocket
Propellant type	Acid-aniline	Oxygen-ethanol
Propellant mass ratio	.54	.70
Average exhaust velocity	6700.	6880 ft/sec
Initial velocity	720.	0 ft/sec
Duration of firing	45.	70 sec
Gross mass	696.	27,900 lbs
Frontal area	0.8	23 sq ft
Mass per unit area	850.	1220 lbs/sq ft
Observed altitude	44.	107 miles
Computed altitude	57.	119 miles

The disagreements in both cases are probably due to such effects as the increase in drag during flight because of slight yawing motions, inefficient utilization of propellant in the starting and stopping periods, etc. In particular, the drag coefficient of the WAC CORPORAL is known to be higher than that given in Fig. 50. The exhaust velocity given for the V-2 rocket has been corrected for the drag acting on the steering vanes located in the jet.

3. Considerations of Flight into Space

The subject of space flight involves many diverse problems such as methods of trajectory control, navigation, communication, take-off and landing techniques, etc. The basic physical question, however, concerns the means for imparting the necessary energy to the vehicle so that it may escape the gravitational grasp of the earth. Although some consideration has been given to the gun-like expulsion of a free piston from a tube, investigators are agreed that the only feasible method is the use of the rocket[36].

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[36] H. Lorenz, "The possibility of space travel", Ver. Deut. Ing. 71, 651-654 (1927) and

F. J. Malina and M. Summerfield, "The problem of escape from the earth by rocket", Proc. Sixth Int. Cong. Appl. Mech. (1946)

The energy or velocity requirements for escape from the earth will be examined, and then the design of rockets capable of producing such velocities will be considered. The following cases are of interest:

i) The earth-satellite, a vehicle permanently revolving in a closed elliptic or circular orbit around the earth, at a distance that is small compared with the radius of the earth.

ii) The "stationary" earth-satellite, a body revolving in a circular orbit with the same angular velocity as the earth so that it always appears overhead to an observer on the earth.

iii) Complete escape from the earth, but with insufficient energy to depart from the solar system.

iv) Complete escape from the earth and the solar system.

An earth-satellite (Case i) revolving in a circular orbit just outside the atmosphere of the earth could remain in such an orbit indefinitely without requiring the expenditure of additional energy, unless struck by a meteoroid[37]. The distance of the orbit

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[37] Fred L. Whipple, "Meteors and the earth's upper atmosphere", Rev. Mod. Phys. 15, 246-264 (1943)

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from the earth should be sufficient to escape the drag of the atmosphere. Its velocity must be such as to balance the gravitational attraction of the earth, according to the following equations:

$$v^2/(R+h) = g_0 R^2/(R+h)^2 \tag{106}$$

$$v = \sqrt{g_0 R^2/(R+h)} \tag{107}$$

The period of one revolution becomes:

$$T = \frac{2\pi(R+h)}{v} = 2\pi\left(1+\frac{h}{R}\right)\sqrt{\frac{R+h}{g_0}} \tag{108}$$

where R is the radius of the earth, h the altitude of the orbit above the earth's surface, and g_0 the acceleration due to gravity at the earth's surface.

The total energy per unit mass that must be imparted to the vehicle to place it in the orbit is the sum of the kinetic and potential energies involved, as follows:

$$E = \frac{v^2}{2} + \int_0^h g_0 \frac{R^2}{(R+x)^2}\,dx = \frac{v^2}{2} + g_0 h \frac{R}{(R+h)} \tag{109}$$

Substituting equation (107) for v,

$$E = \frac{g_o R}{2}\left[1 + \frac{h}{R+h}\right] \qquad (110)$$

The possible contribution of the peripheral velocity of the earth's surface to the required total velocity has been omitted from these equations as being negligible. The variation of the required total energy with altitude is contained in the second term of equation (110), from which it can be seen that the energy of an orbit at 200 miles altitude is only 5 per cent more than one just above the earth's surface. The total energy that must be imparted to the vehicle to place it in the 200 miles orbit is 3.56×10^8 ft/lbs per slug. The orbital velocity is 25,600 ft/sec and the period of revolution is 1.51 hours.

Consideration of a project of this type necessarily raises many interesting questions that are not within the scope of this paper. These include the determination of the optimum trajectory for entering the orbit, the method of guidance or control of the flight path of the vehicle, the means for communicating with the vehicle while in flight, the hazard of destruction by meteoric bodies, the method of subsequent descent, etc. Much work and analysis are required before practical flights can be realized.

An interesting special case of the satellite rocket is the so-called "stationary satellite" (Case ii), the angular velocity of which is identical with the spin velocity of the earth. The vehicle would appear stationary with respect to an observer on the earth. The possibility has been mentioned that a stationary satellite would be a convenient relay point for continuous short-wave communications with about half the surface of the earth. From equations (107), (108), and (110) it can be calculated that the stationary satellite will be located at a distance of 22,400 miles from the surface of the earth, about six earth-radii away, or one-tenth the distance to the moon. Its orbital velocity will be 10,200 ft/sec, and the total energy required to place it in its orbit is 6.29×10^8 ft-lbs/slug.

Another example of interest is the calculation by the same method of the minimum energy that must be imparted to a vehicle to enable it to escape completely from the earth's gravitational influence (Case iii). By means of equation (110) the total energy required is found to be 6.79×10^8 ft-lbs/slug, only slightly more than that of the stationary satellite. This energy corresponds to the familiar "escape velocity" of 7 miles/sec.

As a final case, escape from the earth and the entire solar system may be considered (Case iv). The attractive force of the sun is sufficient to require a considerable expenditure of energy to achieve complete escape. The influence of the planets is negligible since their combined mass is about one-thousandth that of the sun.

The energy per unit mass required to bring a vehicle from the surface of the earth to a position outside the solar system is the sum of the increase in potential energy due to the earth's field plus the increase in potential energy due to the sun's field minus the kinetic energy corresponding to the orbital velocity of the earth. The first and second terms are GM_e/R and GM_s/S where G is the gravitational constant, M_e is the mass of the earth, R is the radius of the earth, M_s is the mass of the sun, and S is the distance between the earth and sun. The third term is $\frac{1}{2}GM_s/S$. The kinetic energy resulting from the earth's spin is neglected. The required energy per unit mass is then given by:

$$E = \frac{GM_e}{R} + \frac{GM_s}{S} - \frac{\frac{1}{2}GM_s}{S} = \frac{GM_e}{R}\left[1 + \frac{\frac{1}{2}(M_s)(R)}{(M_e)(S)}\right] \tag{111}$$

or

$$E = g_o R \left[1 + \frac{\frac{1}{2}(M_s)(R)}{(M_e)(S)}\right], \text{ since } g_o = \frac{GM_e}{R^2} \tag{112}$$

Substituting, $\frac{(M_s)}{(M_e)} = 3.32x10^5$

$$\frac{(R)}{(S)} = 4.26x10^{-5}$$

$$g_o R = 6.79x10^8 \text{ ft-lbs/slug.}$$

Then the value of E becomes $5.48x10^8$ ft-lbs/slug.

In the next two sections the characteristics of rockets capable of attaining the energies required in these four cases will be considered.

4. General Theory of Multiple-Step Rockets

The consideration of multiple-step rockets arises necessarily from the seeming difficulty of ever achieving light enough structure in individual rockets to attain velocities sufficient for escape from the earth. This may not apply, of course, to nuclear energy rockets. In the previous section it was shown that velocities of the order of 25,000 ft/sec or more are required. Taking this figure, the necessary characteristics of escape rockets may be calculated by means of equation (101).

One of the highest exhaust velocities obtainable by means of a chemical reaction is that of the oxygen-hydrogen combination, about 10,500 ft/sec at sea-level (See Table ~~III~~ IV, Part I). It can be assumed, in accordance with the discussion of Part I, Section 10, that the average value of the exhaust velocity, during upward flight out of the atmosphere, may be as much as 20 per cent greater than the sea-level value. This effect is a result of the reduced back pressure prevailing at upper altitudes, and its exact magnitude depends on the operating combustion pressure, the design of the nozzle, and the flight trajectory.

Inserting 12,500 ft/sec for c in equation (101), and assuming a burning time t_p of 100 seconds, it is found that a rocket velocity of 25,000 ft/sec can be obtained only if the propellant mass ratio ζ exceeds .895. That is, the empty mass of the rocket, including the payload, may not exceed 10 per cent of the gross mass. The higher rocket velocities required for cases ii, iii, and iv of the preceding section would necessitate even lighter structure. It is outside the scope of this paper to examine in detail the factors that enter into the structural mass of a rocket, but it should suffice to point to the V-2 rocket which has

ζ equal to .70 to appreciate the difficulty of developing a practical rocket vehicle having ζ of .90 or more.

The velocity attainable with a single-step rocket is severely limited by the fact that propulsion energy must be utilized to accelerate continually the entire empty mass of the rocket even after the major portion of that empty mass is no longer useful. The idea therefore suggests itself that a rocket consisting of several steps be used, each of which operates independently and carries its own propellant load. (Ref. 36) As each step is exhausted it is dropped from the rocket, and the propulsion of the remainder is taken up by the next step. The payload is carried in the last, or smallest step of the rocket (See Fig. 51).

Fig. 51

The performance of multiple-step rockets may be analyzed by the methods derived in Section 2. Let the number of steps equal N, each step being numbered in sequence so that the last step, the Nth, carries the payload of mass M_e. A parameter called the payload ratio, λ[38], can be defined for each step

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[38] The symbol λ was used earlier (Section 9) in an entirely different context to represent the jet divergence factor.

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as the ratio of the mass of the carried load or payload to the mass of the rocket at the moment that step begins to fire.

It has been shown in Ref. (36) that the optimum arrangement is that in which the values of λ for all the steps are equal. For example, in a 3-step rocket, a 100 lb# payload consisting of a radio and instruments would be carried within the body of the third step, and the gross mass of this step, including the payload, might be 500 lbs. The payload ratio for the third step, λ_3, is 0.20. The third step is launched in flight from the second step whose gross mass, including the third step as a payload, is 2500 lbs. The value of λ_2 is also 0.20. The gross mass of the first step, which is really the entire rocket assembly, is 12,500 lbs, so that the mass of the second step constitutes 0.20 of the total. In general, then, this assumption leads to the relation:

$$M_o^{(1)} = \lambda^{-N} M_e \tag{113}$$

where $M_o^{(1)}$ is the initial mass of the first step, in other words, the gross mass of the rocket.

By definition, the gross mass of the first step, $M_o^{(1)}$, includes the mass of the second step, $M_o^{(2)}$, the mass of propellants in the first step, $M_p^{(1)}$, and the empty mass of the first step, $M_e^{(1)}$. In general, for the Nth step, this relation is:

$$M_o^{(n)} = M_o^{(n+1)} + M_p^{(n)} + M_e^{(n)}, \text{ for } 1 \leq n \leq N \tag{114}$$

and $M_o^{(N+1)} = M_e$

It may be further assumed that all the steps in a given rocket can be designed with equal structural effectiveness.

(It is believed that this is not far from the truth, except perhaps for small rockets under 100 lbs mass). This assumption can be expressed by defining a parameter called the structural factor ϵ [39] whose value is the same for each step:

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[39] The symbol ϵ was used earlier (Section 9) to represent nozzle area ratio.

$$\epsilon = \frac{M_e^{(n)}}{M_e^{(n)} + M_p^{(n)}}, \text{ for } 1 \leqslant n \leqslant N \tag{115}$$

The structural factor of the V-2 rocket is 0.24, the lowest value that has been achieved in any rocket vehicle to date.

The propellant mass ratio ζ , defined in Section 2, is related to ϵ and λ in the following manner:

$$\zeta = (1 - \lambda)(1 - \epsilon) \tag{116}$$

Fig. 52 illustrates schematically the various masses involved in the definition of λ , ϵ and ζ .

Fig. 52

According to equation (101) the contribution of each step to the speed of the rocket is given by the following equation:

$$v_n - v_{n-1} = -c_n \ln \left[\epsilon (1 - \lambda) + \lambda \right] - g_o t_n \tag{117}$$

when v_n is the velocity of the rocket at the end of burning of the Nth step, c_n is the average value of the exhaust velocity during the firing of the Nth step, and t_n is the firing period of the Nth step. Because the initial steps of the rocket are generally large enough to reduce the drag-thrust ratio to negligible values, and the lat~~t~~er steps operate above the atmosphere, the drag term is omitted from equation (117). It is assumed that the powered flight is essentially vertical, and that the sea-level value of g may be used for all steps without much error. Each step starts to fire immediately after the preceding step has ceased firing, and is discarded immediately after it is exhausted. A detailed treatment of the validity of the above assumption may be found in Ref. (36).

Insert: The velocity v_N of the rocket at the end of burning of the last step is simply the sum of the contributions given in Eq. (117):

Insert → ($-Nc \ln [\ \ ] - g_0 t_p$)

~~The velocity~~ $$v_N = -Nc \ln \left[\epsilon (1 - \lambda) + \lambda \right] - g_0 t_p \qquad (118)$$

where t_p is the total firing time, and c is the average value of the exhaust velocity over the whole firing period.

The two important equations (113) and (118) can be put in dimensionless form by introducing the overall mass ratio, G, and the velocity ratio S, as follows:

$$G = \frac{M_0^{(1)}}{M_e} = \lambda^{-N} \qquad (119)$$

$$S = \frac{v_N + g_0 t_p}{c} = -N \ln \left[\epsilon (1 - \lambda) + \lambda \right] \qquad (120)$$

The significance of these equations may be clarified by outlining the general approach to the problem of selecting a suitable step-rocket for a given mission. The nature of the mission determines v_N, and the propellant to be used determines the value of c, thus establishing the magnitude of the velocity ratio S. Prevailing experience in the design of structures usually indicated the magnitude of ϵ that may be assumed. With

S and ϵ selected, various combinations of λ and N may be calculated from equation (120), and their simultaneous effect on the required mass ratio G follows from equation (119). In general, the larger the chosen number of steps, the larger is the payload ratio λ and the smaller is the required mass ratio G.

These effects may be examined quantitatively by reference to Figs. 53 and 54, when equations (119) and (120) are plotted

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Figs. 53 and 54

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for several values of S and ϵ. Certain interesting generalizations about step-rockets may be observed:

a) For a given mission, the mass of the required rocket is proportional to the mass of payload to be carried, despite the seeming insignificance of the payload in relation to the large mass of the rocket. If a payload of 100 lbs required a 10,000 lb rocket, a payload of 200 lbs would require a 20,000 lb rocket.

b) For given values of λ and ϵ, the number of steps required is directly proportional to the velocity ratio, and the required gross mass varies exponentially with the velocity ratio or the number of steps. Thus, if a 3-step rocket weighing 1000 lbs is required to achieve a speed of 25,000 ft/sec, with a given payload, then a 6-step rocket weighing 1,000,000 lbs is required to achieve a speed of of 50,000 ft/sec.

c) The required gross mass decreases as the number of steps is increased, but this decrease becomes less pronounced as the number becomes large. Mathematically it may be shown that if ϵ and S are held constant in equations (119) and (120), G approaches an asymptote as N approaches infinity. Actually, the number N should not be chosen too large, lest the complexity and multiplicity of mechanisms cause an unfavorable increase in the structural factor ϵ. It seems, from inspection of Figs. 53 and 54, that a favorable combination of circumstances is obtained when λ lies between 0.20 and 0.40.

In the next section this analysis will be applied to the calculations of step-rockets capable of achieving the missions outlined in the previous section.

5. Numerical Examples of Multiple-Step Rockets

In Section 3 the energy required to escape from the earth was calculated, with reference to four specific missions. In Section 4 a method was presented by which the velocity at the end of burning of a step-rocket may be calculated. By carrying this analysis further it is possible to derive a general expression for the altitude at the end of burning, and thus, an expression for the potential energy of the vehicle at the end of burning. By equating the expression for the total energy to the required energy derived in Section 3, the various characteristics of escape rockets can be computed.

An analysis of this kind is more complicated than is necessary for the purpose of this paper. An approximate expression for the total energy per unit mass of the vehicle can be obtained in the following manner. Assume that the powered portion of the flight is directed vertically upward with an average acceleration of νg_0, and that the force of gravity is constant over the distance of powered flight. It can readily be derived that the gain in kinetic energy is equal to ν times the gain in potential energy. The increase in total energy may then be written as follows:

$$E = \frac{\nu + 1}{\nu} \left[\frac{(v_N + v_0)^2}{2} - \frac{v_0^2}{2} \right] \qquad (121)$$

When v_0 is the initial or launching velocity of the first step and v_N is defined by equation (118). By means of this equation the value of v_N required for a specific mission may be calculated.

The numerical examples in this section will be computed for the case $\nu = 5$, although it is obvious that the precise acceleration is not critical as long as $\nu \gg 1$. Too large an acceleration must be avoided, however, lest the vehicle be damaged by high temperatures caused by atmospheric friction. In the first three cases discussed in Section 3, the initial velocity v_0 is taken to be zero, since merely departure from the earth is considered. In the fourth case of departure from the solar system, v_0 is equal to the orbital velocity of the earth around the sun, 98,000 ft/sec. The values of

v_N calculated from equation (121), and the corresponding burning times and altitudes at the end of burning are listed in Table ~~VIII~~ X.

TABLE ~~VIII~~ X

Characteristics of Four Escape Missions

Mission	Energy Required for the Mission ft-lbs/slug	Velocity (4) at End of Burning ft/sec	Altitude at End of Burning Miles	Duration of Powered Flight Seconds
Earth-satellite at 200 miles (Period = 1.5 hours)	3.56×10^8	25,600	200 (3)	150
"Stationary" Earth-satellite at 22,400 miles	6.29×10^8	32,500	620	200
Complete Escape from the Earth	6.79×10^8	33,700	670	210
Complete Escape from the Solar System (1)	54.8×10^8	39,000 (2)	890	240

NOTES: (1) The large energy corresponding to Case 4 is calculated with respect to the sun: the velocity is calculated with respect to the earth.

(2) To escape from the solar system the velocity of the rocket must be tangential to the earth's orbit to take advantage of the orbital velocity of 98,000 ft/sec.

(3) The powered flight path of Case 1 is not straight upward but curved to enter the orbit.

(4) The velocity at end of burning has been calculated on the assumption that the average acceleration of each step is approximately $5g_o$.

As a first example, an earth-satellite rocket is considered that utilizes the oxygen-hydrogen combination as a propellant. As in Section 4, it may be assumed that the average value of the exhaust velocity during powered flight can be as much as 12,500 ft/sec, (about 20 per cent more than the sea-level value listed in Table ~~III~~ IV, corrected in accordance with the footnote therein). Because of the low average density of the two liquids, (14.8 lbs/ft^3) larger tanks, lines, valves, pumps, etc. are needed for a given weight of propellant than with denser propellants such as oxygen-ethanol. As a result, the structural factor ϵ would probably be greater than that of the V-2 rocket; a reasonable value is 0.33. The values of v_N and t_p are taken from the first line of Table ~~VIII~~ X.

Then,

$$c = 12{,}500 \text{ ft/sec}$$

$$g_o = 32.2 \text{ ft/sec}^2$$

$$t_p = 150 \text{ sec}$$

$$v_N = 25{,}600 \text{ ft/sec}$$

From equation (120):

$$S = \frac{25{,}600 + 150 \times 32.2}{12{,}500} = 2.44$$

Assume $N = 4$ steps. Then, inserting $\epsilon = 0.33$ in equations (119) and (120):

$$\lambda = 0.318$$

and

$$G = 98.0$$

Let the payload consist of instruments and a radio transmitter whose combined mass is 100 lbs. It should be noted again that the mass of the required rocket is proportional to the mass of the payload. Therefore, in imagining such flights, one must think in terms of micro-instruments, micro-radios, and micro-controls. The gross mass of the rocket from equation (113), becomes:

$$M_o^{(1)} = 98.0 \times 100 = 9800 \text{ lbs.}$$

The mass propellant in the initial step is obtained by the use of equation (116):

$$\zeta = (1 - .318)(1 - .330) = .457$$

$$M_p^{(1)} = .457 \times 9800 = 4480 \text{ lbs.}$$

The assumption of equal ϵ, λ and ν for each step, together with equation (96), requires that t_n, the burning time of any step, be equal to that of any other step.

Therefore, burning time of the first step is one-fourth of the total burning time of 150 seconds:

$$t_1 = 37.5 \text{ sec}$$

The thrust of the first step becomes:

$$F_1 = \frac{M_p^{(1)}}{t_1} c = \frac{4480 \times 12{,}500}{32.2 \times 37.5} = 46{,}300 \text{ lbs}$$

The acceleration at take-off:

$$a_1 \text{ (start)} = \frac{46{,}300 - 9800}{9800} = 3.7\ g_o$$

The acceleration at the end of the firing period of the first step:

$$a_1 \text{ (finish)} = \frac{46{,}300 - 5320}{5320} = 7.7\ g_o$$

The approximate size of the rocket may be estimated by first assuming a reasonable value for the average density of the rocket when loaded. Because of the very low density of hydrogen, the overall density is taken here to be only 17.0 lbs/cu ft. Then taking the slenderness ratio (length/diameter) equal to 10:1, regarding the body of the rocket as a cylinder, and ignoring the volume and mass of the fins, the following dimensions are readily calculated:

Length = 42.0 ft. approx.

Diameter = 4.2 ft. approx.

A schematic drawing of this four-step rocket is given in Figure 51.

In the same manner other examples of multiple-step rockets may be calculated, and for comparison of their properties, the essential characteristics are listed in Table XI ~~IX.~~ (~~See Table IX~~)

Table ~~IX~~ XI

In each case, the payload is assumed to have a mass of 100 lbs. The average density of all oxygen-hydrogen rockets is taken to be 17.0 lbs/cu ft, and that of nitric acid-aniline to be 50.0 lbs/cu ft. The structural factor ϵ is estimated to be 0.25 for all acid-aniline rockets and 0.33 for all oxygen-hydrogen rockets. A slenderness ratio of 10:1 is chosen for all cases. The average exhaust velocity of the acid-aniline rockets in flight is estimated at 7600 ft/sec.

In conclusion, it seems quite feasible from the engineering viewpoint to design and construct rockets that are capable of escaping from the gravitational field of the earth. By means of multiple-step rockets it is possible to achieve the tremendous velocities required without necessarily awaiting the development of atomic energy propulsion devices. The great importance

Table ~~III~~ XI Characteristics of Eight Escape Rockets

Example	Mission	Propellant	No. of Steps	Gross Mass (10^3 lb)	Initial Thrust (10^3 lb)	Est. Length (ft)	Est. Diam. (ft)
1	Earth-satellite	Oxy-Hyd	4	9.8	46.3	42	4.2
2	Earth-satellite	Acid-An	6	54.4	251.0	52	5.2
3	Stationary satellite	Oxy-Hyd	5	35.4	160.0	64	6.4
4	Stationary satellite	Acid-An	7	387.0	1670.0	100	10.0
5	Escape earth	Oxy-Hyd	5	49.5	218.0	72	7.2
6	Escape earth	Acid-An	8	425.0	2110.0	103	10.3
7	Escape solar system	Oxy-Hyd	6	119.0	538.0	96	9.6
8	Escape solar system	Acid-An	8	2250.0	9420.0	178	17.8

of high exhaust velocity propellants is to be emphasized, as well as the need for high density. Emphasis must also be placed on the lightening of structures and the careful, economical design of the payloads to be carried. But even these improvements are not absolutely necessary: the present state of rocket technology as embodied in the V-2 Rocket is actually sufficiently advanced for the accomplishment of the task, insofar as propulsion is concerned.

Rockets Utilizing Nuclear Energy

44. Nuclear Energy

The advent of nuclear energy in technically usable amounts with its staggering million-fold increase over the energy release in conventional chemical reactions has aroused especial interest in the application of this new technique to rockets. As pointed out in the introduction, it is necessary in effective rocket design to reduce the amount of mass that must be ejected by the expenditure of tremendous amounts of energy. A rocket of 1000 lbs thrust and 6200 ft/sec exhaust velocity operating at 40 per cent thermal efficiency (this efficiency corresponds to 300 lb/sq in chamber pressure for a rocket at sea level) consumes chemical energy at the rate of approximately 10,000 Btu/sec. A rocket of the same thrust and higher exhaust velocity will consume energy at an even higher rate given by:

$$P = Fa/2\eta_i \qquad (122)$$

where P is the power consumed, F is the thrust, a[40] is the exhaust velocity, and η_i is the thermal efficiency.

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[40] In this section we will use a for exhaust velocity in order to save c for its time honored role as a symbol for the speed of light. They are not to be confused with a for velocity of sound and c for effective exhaust velocity, as used in earlier sections.

It is out of place here to discuss nuclear reactions in detail, however a very clear discussion of the general theory of nuclear reactions is given by Morrison[41], and a concise account of "atomic" energy has been prepared by Tsien[42].

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[41]P. Morrison, Am. J. of Physics 9, 135 (1941)

[42]H. S. Tsien, Jour. of the Aero. Sci. 13, 171 (1946)

Chemical energy results from the rearrangement of electrons (usually only the outermost electrons) as atoms come together to form molecules. The ionization energy of the hydrogen atom is about 13 electron volts, and this may be taken as the order of magnitude of the energy involved in an elementary chemical process. Nuclear energy arises from the rearrangement of the protons and neutrons of atomic nuclei. Now the energy with which a single neutron or proton is bound to a nucleus is about 12 million electron volts, and so we see at once why the nuclear reaction is so much more energetic than the chemical reaction.

Fission

The nuclear reaction which first pointed the way to the technological use of nuclear energy was the fission of U_{92}^{235}. This reaction may be written as follows:

$$n_0^1 + U_{92}^{235} \longrightarrow \text{Primary fission products} + 180 \text{ Mev}$$

$$\text{Primary fission products} \longrightarrow \text{Final products} + 20 \text{ Mev}$$

We may note two facts about this process. First of all the neutron which induces the fission of uranium does not have to carry any kinetic energy into the uranium nucleus. Merely inserting the neutron into the nucleus introduces an excitation energy of 6.4 Mev and only 5.3 Mev is required for fission to be possible. This is important, because it is only slow neutrons which have a large probability of being captured by the uranium nucleus. Secondly, the fission products are not unique, but are distributed over a variety of nuclear species which fulfill the requirements of conservation of charge of mass number. Thus the primary products from the fission of one uranium nucleus might be $I_{53}^{141} + Y_{39}^{95}$ and from another nucleus may be $2Pd_{46}^{118}$. Furthermore the primary fission products are not stable, but have an excess of neutrons as compared to a "normal" nucleus of the same charge and so decay by neutron emission or electron emission until a stable nucleus is formed.

For definiteness we will now write down the fission reaction which seems to be the most probable:

$$n_o^1 + U_{92}^{235} \rightarrow (U_{92}^{236}) \rightarrow 2n_o^1 + I_{53}^{140} + Y_{39}^{94} + 179 \text{ Mev}$$

$$I_{53}^{140} \rightarrow Ce_{58}^{140} + 5e_{-1}^{o} + 17 \text{ Mev}$$

$$Y_{39}^{94} \rightarrow Zr_{40}^{94} + e_{-1}^{o} + 4 \text{ Mev}$$

Where the decay of I to Ce goes through intermediate steps Xe^{140}, Cs^{140}, Ba^{140}, La^{140}.

For calculation we will usually assume that:

$$n_o^1 + U_{92}^{235} \rightarrow 2Pd_{46}^{118} + 170 \text{ Mev}$$

$$Pd_{46}^{118} \rightarrow n_o^1 + Sn_{50}^{117} + 4e_{-1}^{o} + 15 \text{ Mev}$$

On this basis the kinetic energy of a Pd nucleus (since conservation of momentum with equal masses implies equal distribution of energy) will be 85 Mev and the particles will have a velocity of

1.2×10^9 cm/sec or 3.94×10^7 ft/sec.

45. Relativistic Mechanics of Rockets

Ackeret[43] has discussed the application of relativistic mechanics

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[43] J. Ackeret, Helv. Phys. Acta 19, 103 (1946)

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to rockets. Although the velocity of fission fragments is generally less than 3×10^9 cm/sec, the velocity at which relativistic effects first become important, nevertheless it is interesting to see how the laws of relativity will modify the behavior of the "classical" rocket. Our investigation will also make clear-cut the domain in which classical theory may safely be used.

a) Static Thrust

The most simple modification of the rocket equation occurs when we consider the static thrust from a rocket with an exhaust velocity, a, which is near the speed of light, c. The usual relativistic definition of force as rate of change of momentum gives for the thrust (in vacuum):

$$F = m_o a(1 - a^2/c^2)^{-1/2} \tag{123}$$

where m_o is the rate of mass flow from the rocket as observed from a system in which the rocket is at rest.

The thrust given by equation (123) is higher than the thrust, $F = m_o a$, of the classical rocket by the Lorentz factor. The relativistic thrust is 100 per cent greater than the classical thrust when the ratio a/c is 0.865, it is 10 per cent greater when the exhaust velocity, a, is 0.415c, and if a is only 0.10c then relativistic thrust is only one-half per cent larger than the classical thrust.

b) The Rocket Equation

From the point of view of static thrust the relativistic effects seem to be all to the good, for the thrust at a given mass flow is increased. However, consideration of static thrust is inadequate to describe the propulsion of a rocket. Now, not only will the Lorentz increase of mass with velocity increase the mass of the exhaust jet, but also it will increase the mass of the rocket. Therefore, the picture is not as simple as it might seem at first glance. Ackeret has found a clever scheme for calculating the final velocity of a rocket as observed in a coordinate system in which it was initially at rest, in terms of the ratio ζ of propellant weight to initial gross weight and in terms of the exhaust velocity a relative to the rocket. This will now be presented.

Consider a rocket in empty space without the influence of external forces or fields. We will observe the motion from a system of coordinates in which the rocket was initially at rest. Let us assume that mass is ejected to the right with a velocity u_2, and the rocket proceeds to the left with a velocity u_1. The rest mass of the rocket at any instant will be m_{01}, and the rest mass of a small portion of the exhaust jet will be dm_{02}. This is indicated in Figure 55. To save writing let the Lorentz factors

Fig. 55

relative to our coordinate system be:

$$k_{1,2} = \left[1 - u_{1,2}^2/c^2\right]^{-1/2} \tag{124}$$

where c is the velocity of light.

The equations of motion of the system are as follows:

a) The conservation of energy:

$$d(m_{01}k_1c^2) = k_2c^2dm_{02} \tag{125}$$

b) The conservation of momentum:

$$d(m_{01}u_1k_1) = k_2u_2dm_{02} \tag{126}$$

c) The relativistic formula for addition of velocities:

$$u_2 = (a-u_1)(1-u_1a/c^2)^{-1} \tag{127}$$

where a is the (constant) exhaust velocity relative to the rocket.

If one eliminates k_2dm_{02} between equations (125) and (126) there is obtained:

$$u_2d(m_{01}k_1) - d(m_{01}k_1u_1) = 0 \tag{128}$$

Eliminating u_2 from (128) by use of (127) we find:

$$(a-u_1)d(m_{01}k_1)+(1-u_1a/c^2)d(m_{01}k_1u_1) = 0 \tag{129}$$

This last expression may be simplified without too much difficulty to give:

$$dm_{01}/m_{01} = -(1/a)du_1/(1-u_1^2/c^2) \tag{130}$$

Let v be the final velocity of the rocket. If $W_p + W_o$ is the gross weight of the rocket before firing and W_o is the final weight of the rocket after all the propellant has been ejected, then the ratio of propellant weight to initial gross weight is:

$$\zeta = W_p/(W_o+W_p) \tag{131}$$

Integration of equation (130) then gives

$$(1 - \zeta) = \left\{\frac{c - v}{c + v}\right\}^{c/2a} \tag{132}$$

or

$$v/c = \left[1 - (1 - \zeta)^{2a/c}\right] \Big/ \left[1 + (1 - \zeta)^{2a/c}\right] \tag{133}$$

This may be compared (Cf. equation 101) with the classical equation for field-free space.

$$v/a = \log(1 - \zeta) \tag{134}$$

The general nature of these results is shown in Figure 56.

Fig. 56

We see that the critical case $\zeta = 1$ corresponds to the velocity of light in the relativistic theory and to an infinite velocity in the classical theory.

Several things are noteworthy in Figure 56. First of all, if $a = c/10$, even for ratios ζ very near 1.0 there is almost no difference between the classical and relativistic performance of rockets. For an exhaust velocity of half the speed of light an appreciable difference between relativistic performance and classical performance occurs at mass ratios as low as $\zeta = 1/2$. The limiting case, $a = c$, the highest possible exhaust velocity which is allowed in the theory of relativity has been shown since it may be of some interest. In general, the relativistic correction to classical theory is such as to reduce the terminal velocity of the rocket.

46. Problems in the Utilization of Nuclear Energy

We have seen that the fission of U^{235} will produce particles with the velocity of 1.2×10^9 cm/sec. This velocity is less than one-tenth the velocity of light, and so we need only consider classical mechanics since the study of relativistic mechanics showed that even with exhaust velocities one-tenth that of light classical mechanics would be reliable up to mass ratios very near one.

a) The Ideal Rocket

Assume that we have a rocket consisting of a main body covered on the rear by a layer of fissionable material. In order to simplify the argument we assume that when the material reacts half of the particles go directly to the rear in the form of an exhaust jet and the other half go directly forward and transmit their momentum to the rocket. Since there is no known method of "reflecting" high speed fission fragments, we must assume that the forward moving particles eventually stop in the main body of the rocket. We will assume that the mass of the main body of the rocket is large as compared to the mass propéllant, which will shortly be seen to be true.

The conservation of momentum then gives:

$$Mv = (m/2)a \qquad (135)$$

where M is the mass of the main body, n is the total mass of fissionable material, v is the final velocity of the rocket, and a is the exhaust velocity, 1.2×10^9 cm/sec in our example. Suppose we let M be one ton and v be the velocity necessary to escape from the earth which is 1.12×10^6 cm/sec. Then we find for m the value:

$$m = 3.75 \text{ pounds}$$

This is a small value, less than two-tenths of one per cent of the mass of the rocket. Nuclear energy apparently reduces the magnitude of the mass ratio required for a one-step rocket to escape from the earth from $\zeta = 0.96$ for the hydrogen-oxygen chemical propellants to $\zeta = 0.00187$ for uranium. However there is a difficulty.

We recall that half the fission products were stopped in the body of the rocket, carrying momentum <u>and</u> energy. The amount of energy the one ton of rocket must absorb is:

$$E = (\tfrac{1}{2})\tfrac{1}{2}ma^2 \qquad (136)$$

For the values of m, a, given above this is:

$$E = 5.5 \times 10^{10} \text{ Btu}$$

Assuming that the rocket has the rather high specific heat of unity, the main rocket body will attain a temperature of 2.8×10^7 degrees F. A fantastic cooling problem! This situation is aggravated when we realize that the primary fission products which have stopped in the body of the rocket are subject to radioactive decay with the release of approximately 10^9 additional Btu. Of course the actual temperature developed with an arrangement of this sort will be much less, since the specific heat of steel for example, normally only 0.1 will be greatly increased after the material is vaporized.

b) <u>Possible Exhaust Jets</u>

We have just seen that the direct application of nuclear energy to rocket propulsion does not seem to be practical. This brings up the question of a suitable technique for application of this new energy. A good place to start is in the investigation of what will constitute a satisfactory exhaust jet. Assuming that nuclear energy can be controlled in a flexible manner, then one may conceive of rockets utilizing photon (light) jets, jets of charged particles, or jets consisting of particles which are neutral.

(1) The Photon Rocket

A great deal would need to be done to develop intense sources of radiation before the photon-jet would be practical. Although a low-thrust jet would enable a rocket ultimately to attain any desired speed in field free space, near the earth the thrust would have to be at least as great as the weight of the rocket. A simple example makes the difficulty of this clear.

Suppose that the rocket "motor" consists of a layer of incandescent material at the rear of the rocket. We will soon see, if excessive temperatures are to be avoided, that this layer must be large in area, probably forming a disc many times the diameter of the rocket. We assume that the radiation leaves this disc uniformly in all directions contained in the solid angle of 2 at the rear of the disc. Suppose that in a thin layer next to the surface black body radiation exists. The pressure p of this radiation will be given by[44]:

$$p = (1/3)bT^4 \qquad (137)$$

[44] Tolman: Relativity, Thermodynamics, and Cosmology; Oxford Univ. Press, (1934) 139

Here T is the temperature on the absolute scale and the constant b has the value in c.g.s. units of:

$$b = 7.62 \times 10^{-15} \text{ ergs cm}^{-3}(\text{degK})^{-4}$$

Now in this layer the density of energy ρ will be:

$$\rho = bT^4 \tag{138}$$

and the energy will flow away from the rocket with the speed of light c. Consequently the power P radiated from a unit area is:

$$P = bT^4c \tag{139}$$

The rate of weight flow is obtained by dividing the power by c^2/g. The results of a calculation based on these equations are given in Table XII.

TABLE XII

Estimated performance of the radiating disc propelling a "photon-rocket"

Pressure p lb/sq ft	Surface temperature T deg.K	deg.R	Power radiated Btu/sec sq ft	Rate of weight flow lb/sec sq ft
5.32×10^{-2}	10,000	18,000	2.22×10^{5}	5.3×10^{-9}
10	37,000	67,000	3.81×10^{7}	10^{-6}
100	66,000	120,000	3.81×10^{8}	10^{-5}
1000	117,000	210,000	3.81×10^{9}	10^{-4}
2120*	141,000	254,000	8.07×10^{9}	2.1×10^{-4}
10,000	208,000	375,000	3.81×10^{10}	10^{-3}

*Atmospheric pressure

These results emphasize the tremendous power required to obtain a given thrust. A 1000-lb thrust photon rocket consumes 400,000 times as much power as the corresponding rocket utilizing chemical propellants (see equation 122). On the other hand the mass flow of the photon rocket is greatly reduced, the specific impulse being 10^7 sec as compared to 200 sec for conventional rockets. Thus a given mass of "pure photon propellant" could support its initial weight in the gravitational field of the earth for 115 days as compared to 3.3 minutes for the usual chemical propellants.

The difficulty with the photon rocket is the large surface temperature required in order to obtain a reasonable amount of thrust from one square foot of radiating surface. A radiation pressure of only 10/lbsq ft required a surface temperature of 67,000 deg. R. To obtain atmospheric pressure, the temperature must be 254,000 deg. R. The tungsten filament lamp operates at a temperature of approximately 3000 deg. K (tungsten melts at 3655°K). Suppose we are very optimistic and assume it will be possible to construct a radiating disc to operate at a surface temperature of 10,000°K. Using the corresponding radiation pressure in Table XII, we find a surface of 37,400 sq ft is required to support one ton of rocket in the gravitational field of the earth. This is a circular disc 218 feet in diameter. It would seem unlikely that such a large surface could be constructed to operate at high temperature and yet weigh less than one ton, and of course there would have to be carried the power generating apparatus and the payload. The "photon rocket" is not likely to be practical in the near future.

(ii) Jets of Charged Particles

The excellent techniques for accelerating charged particles which have been developed in recent years indicate that a jet of electrons or protons with "exhaust velocities" approaching the velocity of light should be feasible. However, the rapid charging of the rocket would stop a jet contianing only one sign of charge in a very short time. Positive and negative jets could be operated simultaneously, perhaps, so as to overcome this difficulty. However, the energy required to separate the raw "fuel" into ions suitable for acceleration away from the rocket would be rather large, and this energy would be wasted. At the present time the intensity of the beams of charged particles from the known accelerators is far too small to furnish any appreciable thrust.

(iii) Neutral Exhaust Jets

The preceding discussion has narrowed down the practical techniques for the application of nuclear energy to rocket propulsion to an arrangement in which a working fluid is heated by the nuclear reaction. This working fluid would then be used in the conventional exhaust nozzle to supply rocket thrust. Hydrogen might be a suitable material to use as a working fluid because of its low atomic weight. The working fluid technique would also make it possible to get rid of some of the radioactive products of the nuclear reaction by having them carried off in the exhaust jet.

c) <u>The Technique of a Working Fluid</u>

There is a fundamental difference between the conventional rocket utilizing chemical propellants and a rocket propelled by the combination of nuclear energy plus working fluid. In the case of the rocket utilizing chemical propellants, a high performance rocket (for example a rocket which is to leave the earth) must operate at a mass ratio ζ very near unity. In fact for a rocket utilizing the very energetic hydrogen-oxygen chemical reaction this ratio must be 0.96 if a single stage rocket is to escape from the earth. This ratio of propellant weight to initial gross weight is so high that it is not practical to construct such a rocket. One then is forced to build a multi-stage rocket or seek even more energetic fuels. The fundamental cause of this difficulty is that with chemical rockets of this type the propellant is used both as a source of energy and as the mass to be ejected in obtaining thrust. It is true that one may add inert material to the exhaust jet thus making energy supply and mass flow somewhat independent of one another, but since practical exhaust velocities obtainable from chemical propellants are not great enough for high performance rockets this is a purely academic procedure and is not used except in a very minor way to overcome cooling difficulties, (see Sec. 32).

With the system nuclear energy plus working fluid the situation is much different. It turns out that there is an optimum amount of working fluid, and too little working fluid is disadvantageous! This may be seen as follows: The fission reaction is so energetic that it represents, when compared to the chemical scale, almost pure massless energy. Thus the energy released by one pound of U^{235} in the primary fission process is $3x10^{10}$ Btu, while the very energetic hydrogen-oxygen reaction generates only $6.85x10^{3}$ Btu per pound. We saw, in considering the "ideal rocket", that only 3.75 pounds of U^{235} was needed to project a one-ton weight from the earth. These 3.75 pounds are only 0.19 per cent of the total mass of the rocket. Thus in our following considerations we will be quite justified in neglecting the mass of uranium as compared to the mass of the working fluid when considering mass flow. Of course the mass of uranium will be quite important in considerations of energy supply and exhaust velocity.

We now consider a rocket consisting of a payload of weight W_o, uranium of weight W_u, and working fluid of weight W_p. Except for the transformation of part of the mass of uranium into energy[45] nonrelativistic mechanics can be used.

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[45] This statement is not rigorously correct, but it is used to convey the essential notion without circumlocution. See E. F. Barker; American Journal of Physics; **14**, 309 (1946). The correct statement is that part of the mass-energy of the uranium is changed from mass, a sort of potential energy, into energy in a more useful form. Of course the energy produced still has the same mass as it had before the transformation. No mass-energy really appears or disappears.

In a coordinate system at rest with respect to the rocket a small fraction δ of the mass of the uranium is converted into energy, $\delta W_u c^2$, given by Einstein's equation. This energy must be equal to the kinetic energy of the exhaust jet. Thus we obtain:

$$\delta W_u c^2 = \tfrac{1}{2} W_p a^2 \tag{140}$$

For the uranium reaction $\delta = 0.000731$, that is less than one-tenth of one per cent of the mass of uranium is converted into energy. Recall that c is the speed of light and a is the exhaust velocity (we assume that there is no external pressure so that the exhaust velocity a corresponds to the full energy).

Neglecting the weight of uranium we can write the weight (or mass) ratio ζ as:

$$\zeta = \frac{W_p}{W_o + W_p} \tag{141}$$

We may solve equation (140) for a and substitute this result and equation (141) into equation (134) which gives the final velocity v of the rocket in field free space. This gives:

$$v = -c \left[2 \delta W_u / W_p\right]^{\frac{1}{2}} \log \left(1 - \frac{W_p}{W_o + W_p}\right)$$

or (142)

$$v = c \left[2 \delta W_u / W_p\right]^{\frac{1}{2}} \log \left(1 + \frac{W_p}{W_o}\right)$$

Now it happens that if W_o and W_u are held fixed (fixed payload and fixed amount of energy) that v has a maximum with respect to W_p at $W_p \overset{\circ}{=} 4 W_o$. This arises because W_p appears both in the expression for the exhaust velocity (the square root) and in the mass ratio. This maximum is very flat and any value for the mass ratio ζ larger than 0.5 puts one very near the peak. If we consider the one ton of payload and 3.75 pounds of uranium found in the case of the "ideal rocket" and add four tons of working fluid to the system we find a final velocity v for the payload of 4.0×10^{7} cm/sec which is considerably larger than the 1.12×10^{6} cm/sec attained without the use of working fluid.

It is interesting to note that the value of 0.8 for ζ which leads to maximum terminal velocity v is independent of the amount of energy supplied by the uranium (that is, of $\delta W_u c^2$) so long as W_u is small relative to W_0 and W_p. The assumption of small W_u is inherent in equation (141), and will always be valid so long as excessive temperatures have to be avoided. Of course, v will be smaller with less energy, but for a given energy v will have a maximum for $\zeta = 0.8$.

It seems appropriate to indicate here what exhaust velocities might be obtained with the system of nuclear fission plus hydrogen gas as a working fluid taking into account the limitations imposed by temperature. If we take into account the dissociation of hydrogen molecules into monatomic hydrogen and assume that the exhaust is discharged into a vacuum then we may expect to obtain an exhaust velocity of 21,300 ft/sec if the gas can be heated to 4940°F and a velocity 37,300 ft/sec at 10,340°F. This latter exhaust velocity is quite sufficient for a single stage rocket to escape from the earth with a mass ratio of 0.5.

One notes that the exhaust velocity increases more rapidly than $\sqrt{T}$. This favorable result is due to the dissociation of the gas.

Legends for Figures. Part IV

Fig. 42 The original liquid oxygen-gasoline rocket of R. H. Goddard, which achieved 7500 ft altitude in 1935.

Fig. 43 The WAC CORPORAL sounding rocket being loaded into the launching tower.

Fig. 44 The flight of a solid propellant rocket used in the preliminary development of the WAC CORPORAL sounding rocket. The smoke trail shown is about 3 miles long.

Fig. 45 A German V-2 rocket just after take-off at White Sands Proving Grounds, New Mexico.

Fig. 46 Official British press photograph of the German V-2 rocket, dated Dec. 8, 1945. The numbers refer to minor structural details.

Fig. 47 The forces acting on a rocket in vertical flight through the atmosphere.

Fig. 48 The effect of propellant mass ratio and effective exhaust velocity on the altitude attained by a sounding rocket in vertical flight. The influence of air drag has been neglected, and it has been assumed that $\mu = \infty$, t_p - 30 sec, and $h_o = v_o$ - 0.

Fig. 49 The effect of thrust duration t_p and gross mass per unit cross-section μ on altitude of vertical rocket flight. It has been assumed that $\zeta = 0.70$, $c = 7700$ ft/sec, $v_o = h_o = 0$, and that C_d is given by Fig. 50.

Fig. 50 Curve of drag coefficient versus Mach number used in calculating rocket trajectories. It is assumed here that no jet is acting, the angle of attack is zero, and that standard NACA atmospheric tables are applicable.

Fig. 51 Diagram of one possible arrangement of components in a four-step rocket.

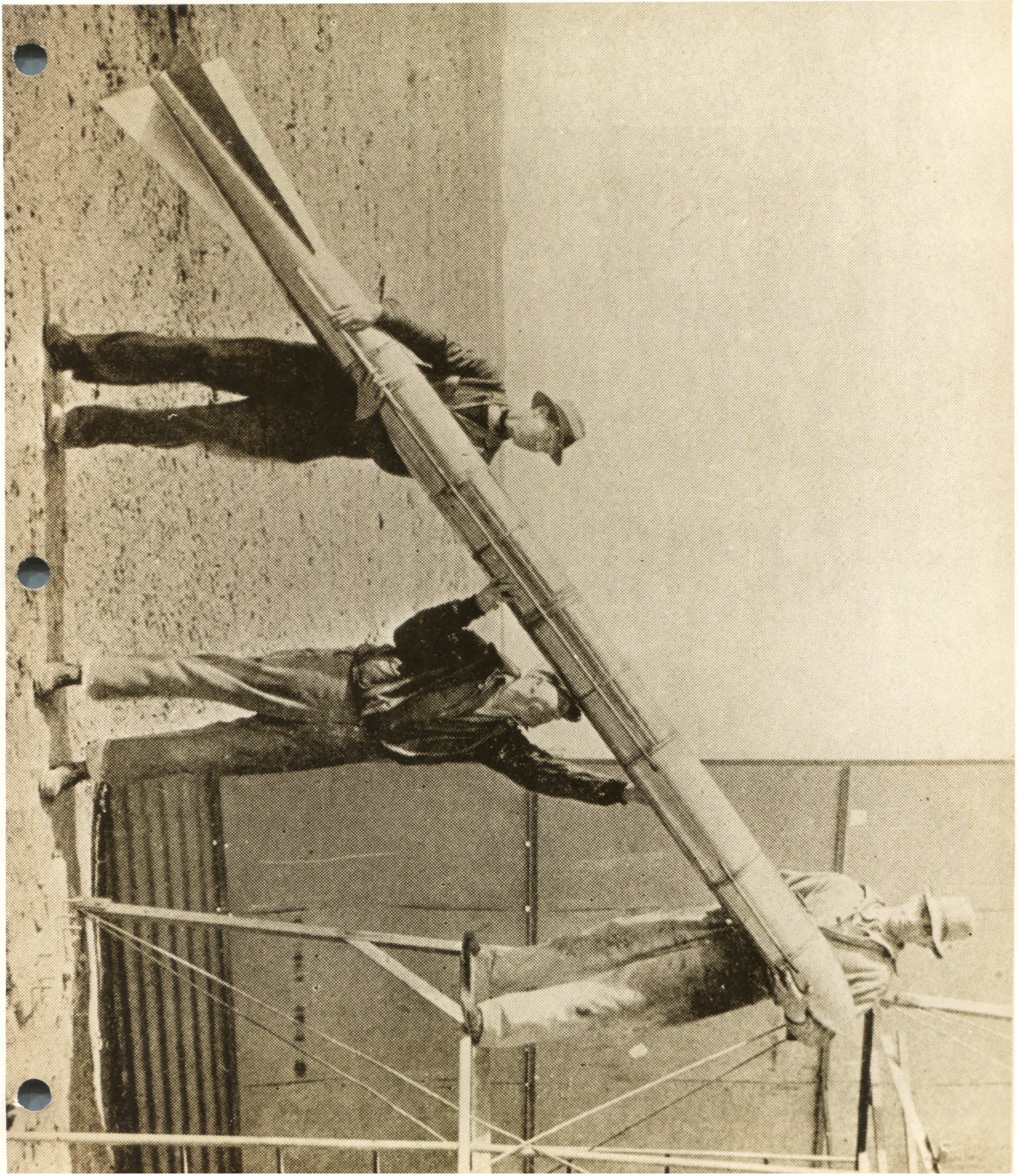
Fig. 52 Diagram of the disposition of masses in a step rocket which enter into the definitions of the propellant mass ratio ζ, the structural efficiency factor ϵ, and the payload ratio λ.

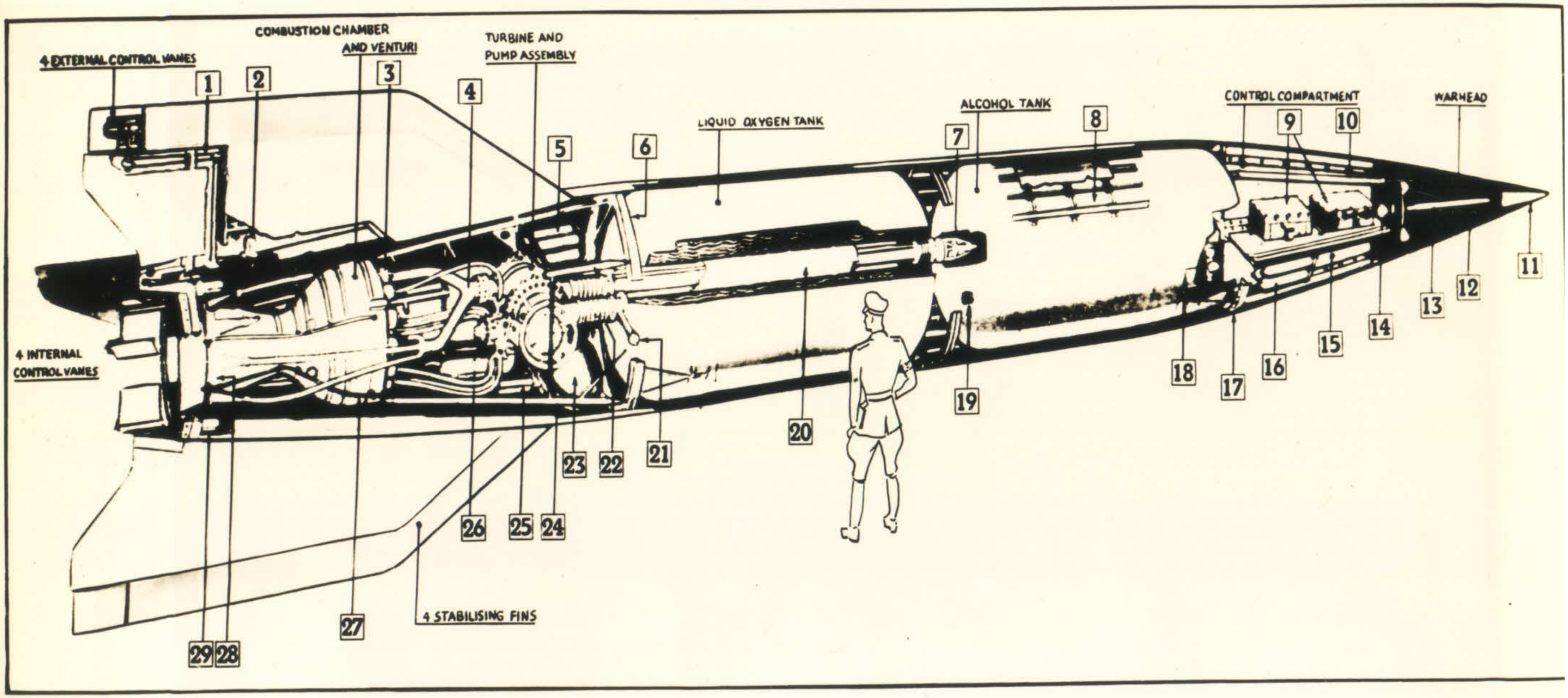
Fig. 53 Graph of the step-rocket payload ratio λ and over-all mass ratio G showing the improvement in λ and G as the number N of steps increases. Two typical velocity ratios S (3 and 6) have been chosen, and a structural efficiency factor $\epsilon = 0.25$.

Fig. 54 Graph of payload ratio λ and over-all mass ratio G similar to Fig 53, but with an improved structural efficiency factor $\epsilon = 0.20$.

Fig. 55 Coordinate system and notation used in discussing the relativistic motion of a rocket. These axes are fixed relative to the initial ~~vert~~ rest position of the rocket.

Fig. 56 The terminal velocity v of a rocket in field free space (expressed as a dimensionless ratio to the fixed velocity of light c), plotted as a function of propellant mass ratio ζ and exhaust velocity a for classical and relativistic motion. a is expressed as a fraction of the velocity of light c.





4 EXTERNAL CONTROL VANES
COMBUSTION CHAMBER AND VENTURI
TURBINE AND PUMP ASSEMBLY
LIQUID OXYGEN TANK
ALCOHOL TANK
CONTROL COMPARTMENT
WARHEAD
4 INTERNAL CONTROL VANES
4 STABILISING FINS
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2
3
4
5
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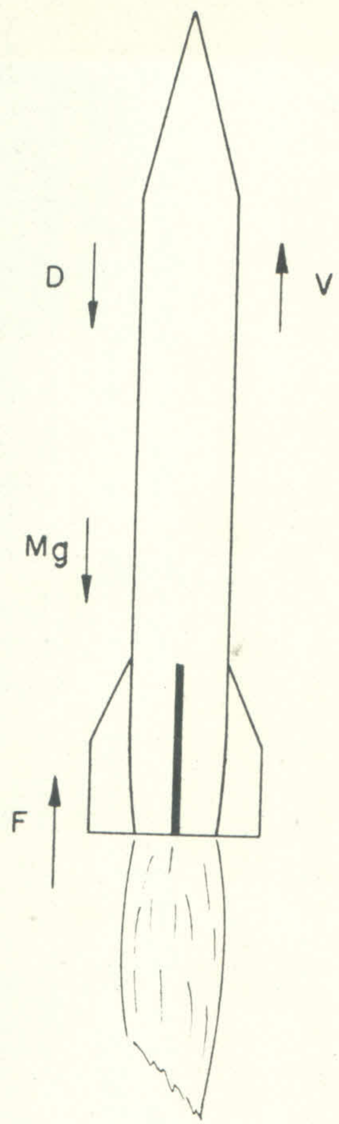

Fig. 47
H. S. Seifert

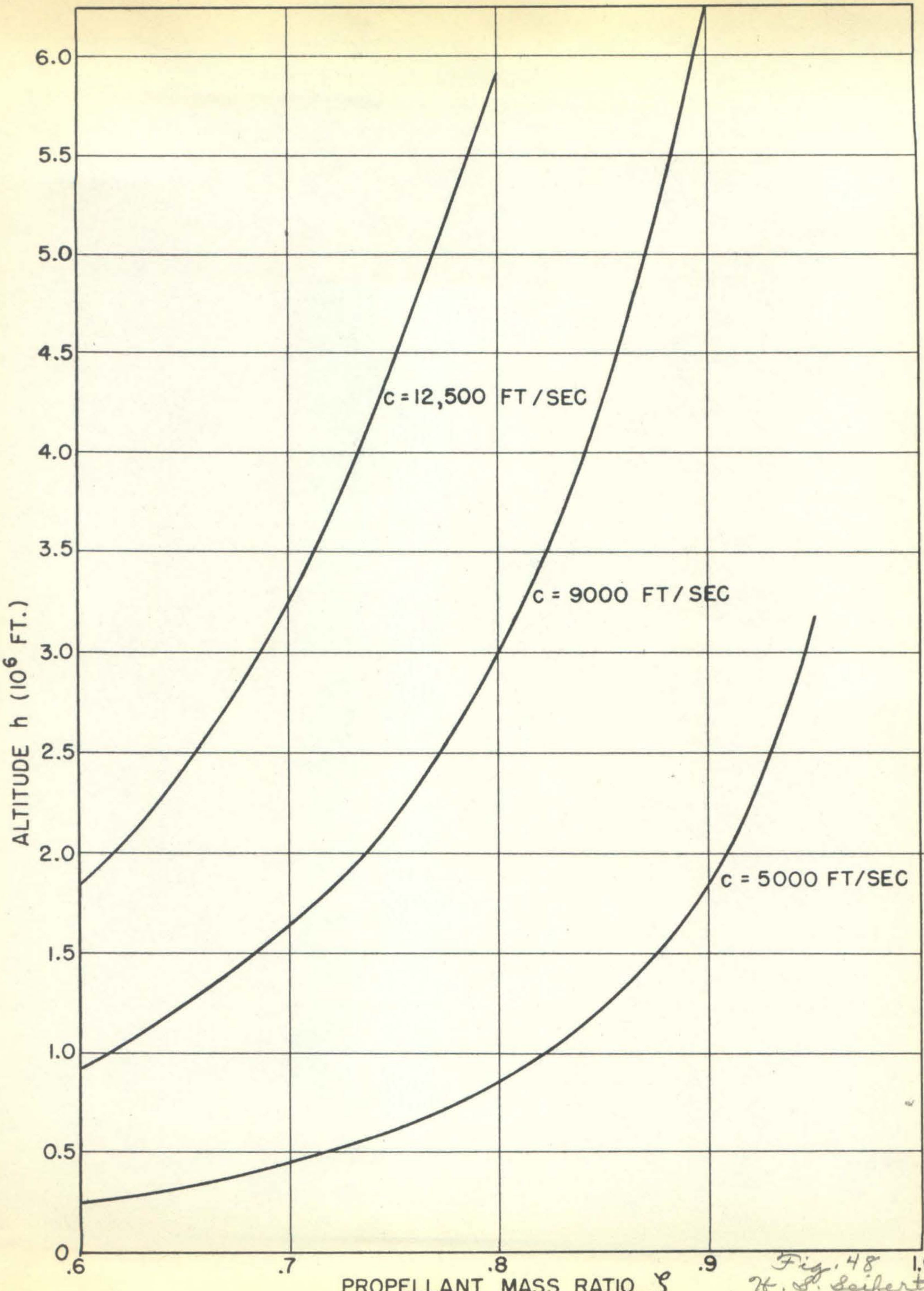

Fig. 48
H. S. Seifert

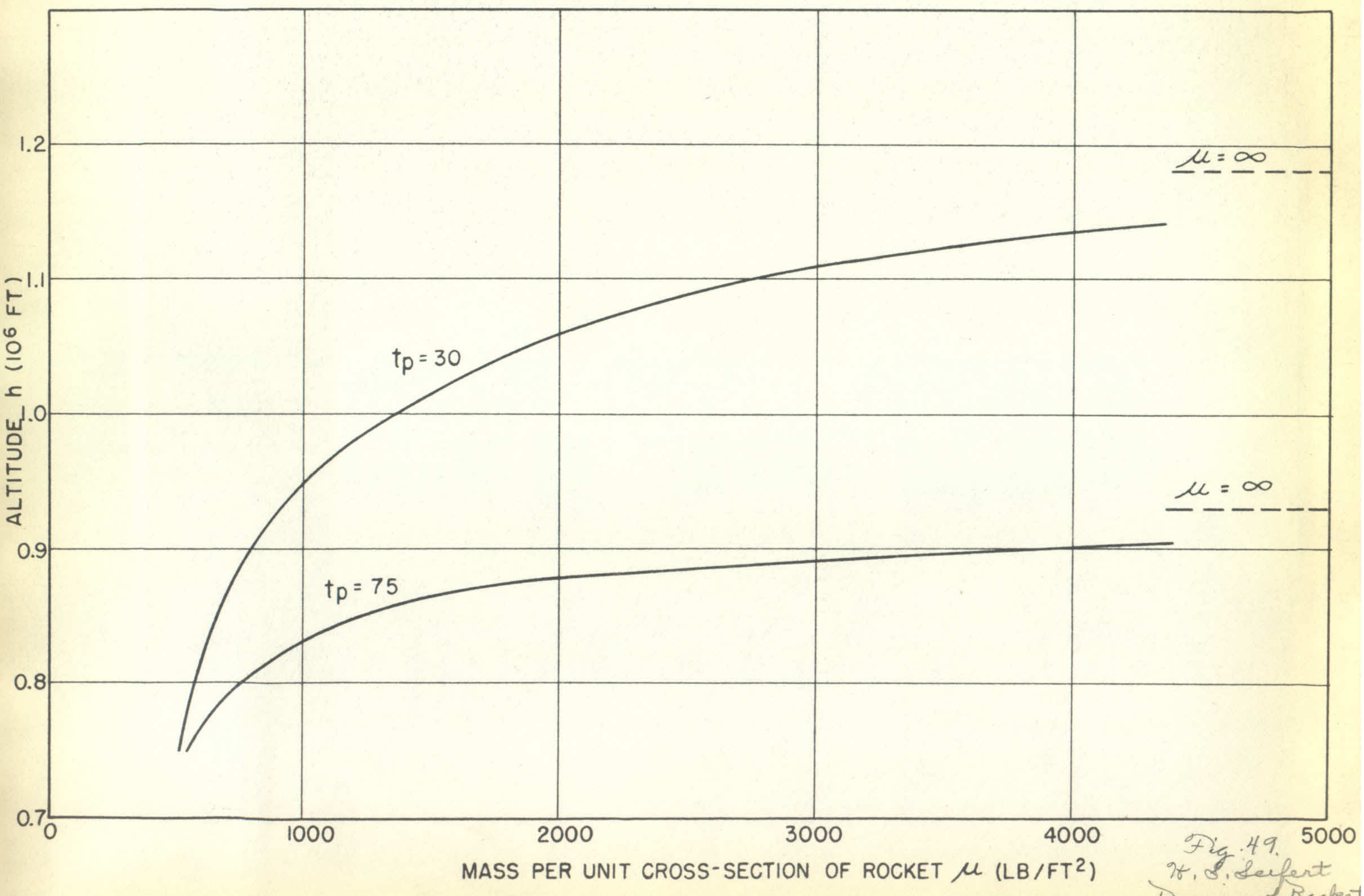

Fig. 49.
H. S. Seifert
Physics of Rockets

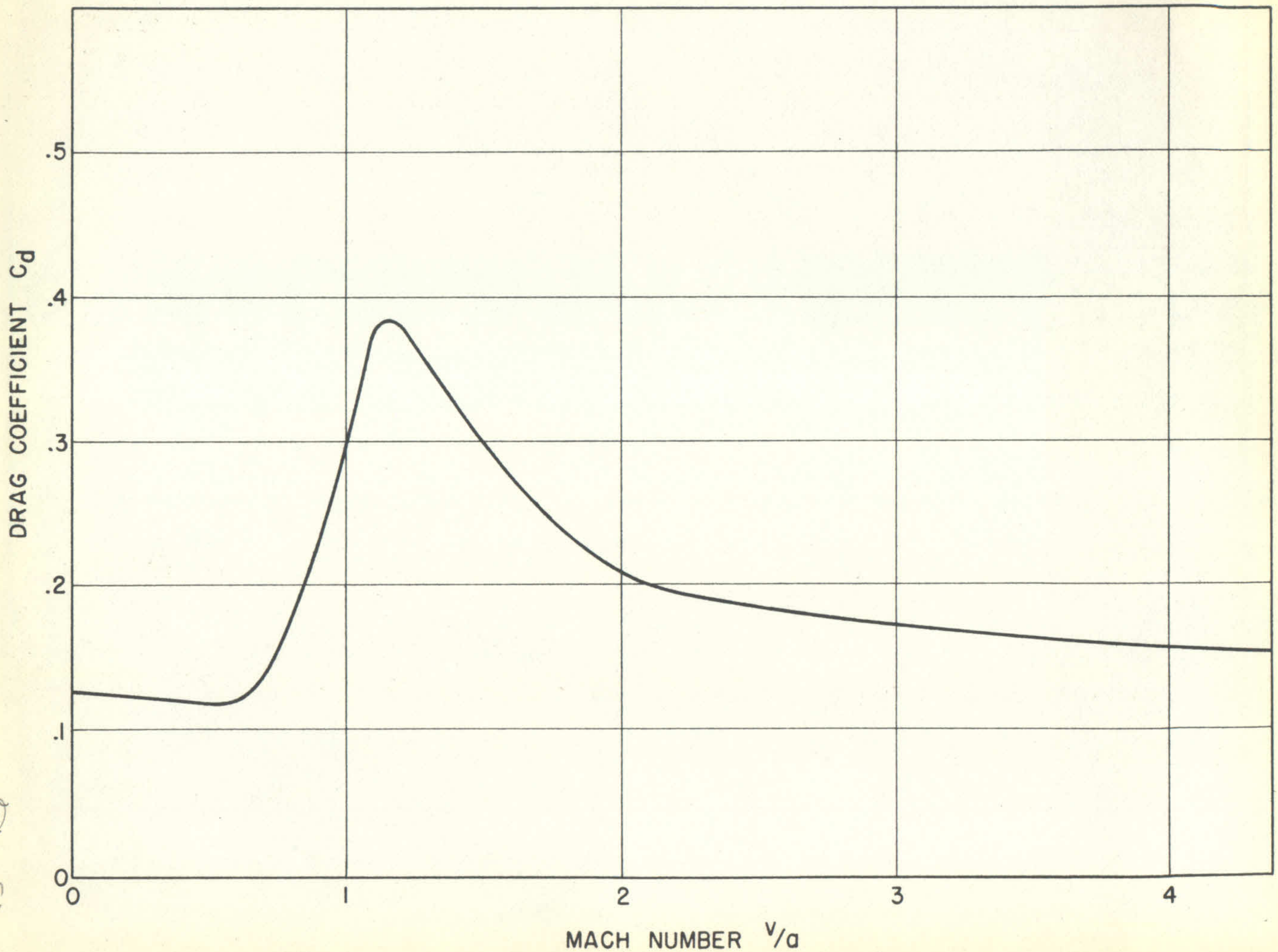

Fig. 50
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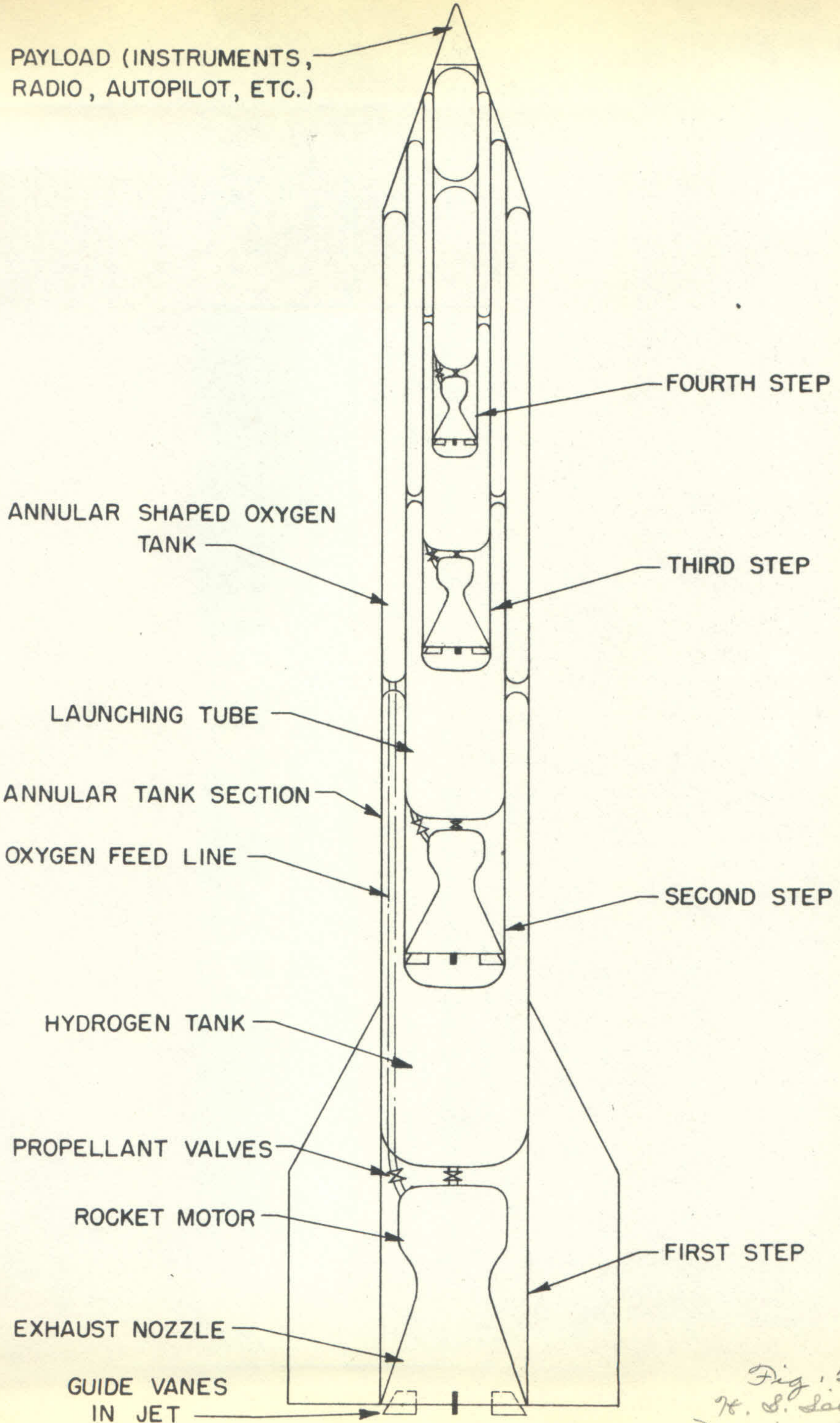
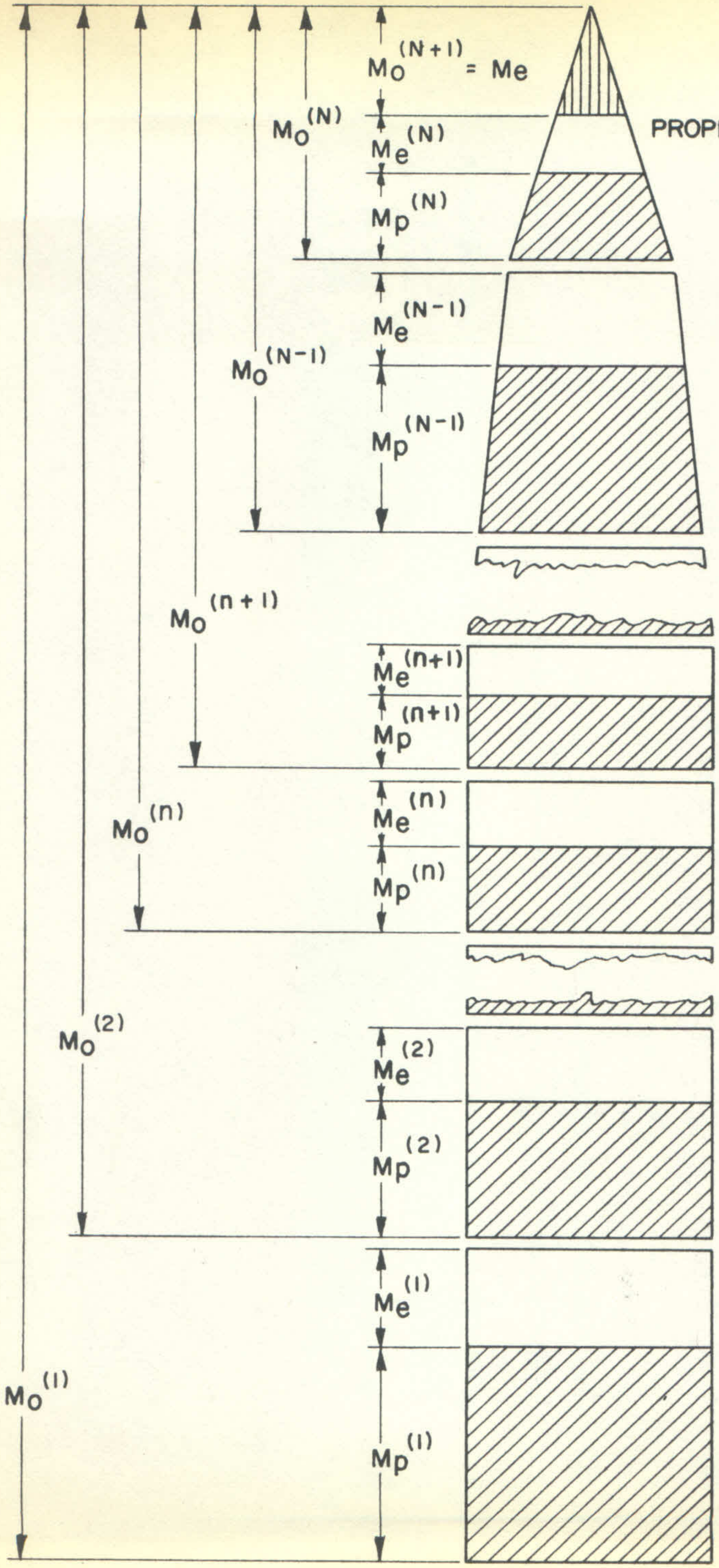


Fig. 51
H. S. Seifert



PROPELLANT MASS

$$\lambda = \frac{M_o^{(2)}}{M_o^{(1)}} = \frac{M_o^{(n+1)}}{M_o^{(n)}} = \frac{M_e}{M_o^{(N)}}$$

$$\epsilon = \frac{M_e^{(1)}}{M_e^{(1)} + M_p^{(1)}} = \frac{M_e^{(n)}}{M_e^{(n)} + M_p^{(n)}}$$

$$\zeta = \frac{M_p^{(1)}}{M_o^{(1)}} = \frac{M_p^{(n)}}{M_o^{(n)}} = \frac{M_p^{(N)}}{M_o^{(N)}}$$

$$\zeta = (1-\lambda)(1-\epsilon)$$

Fig. 52
H. S. Seifert
Physics of Rockets

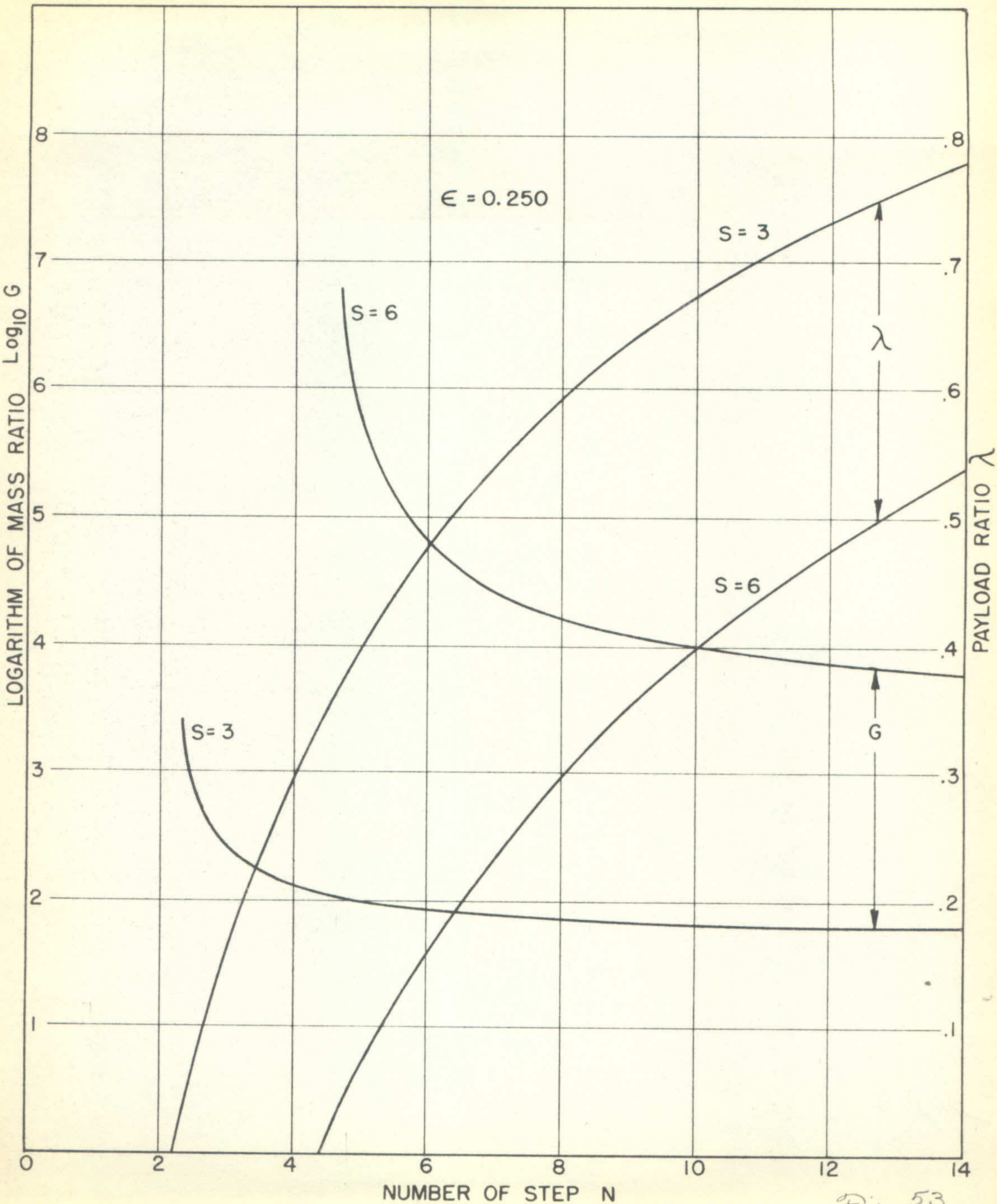


Fig. 53
H. S. Seifert

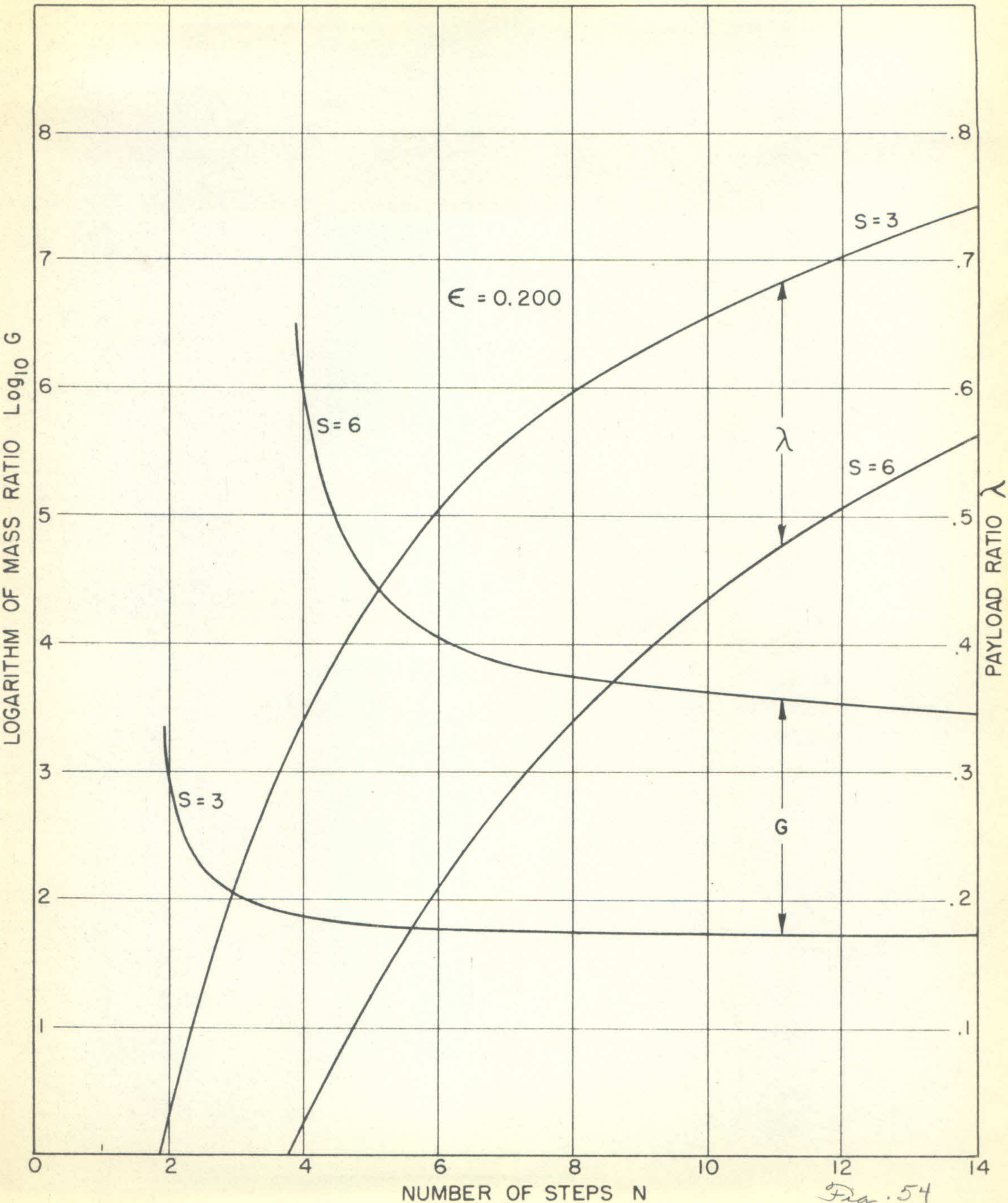


Fig. 54
H. S. Seifert

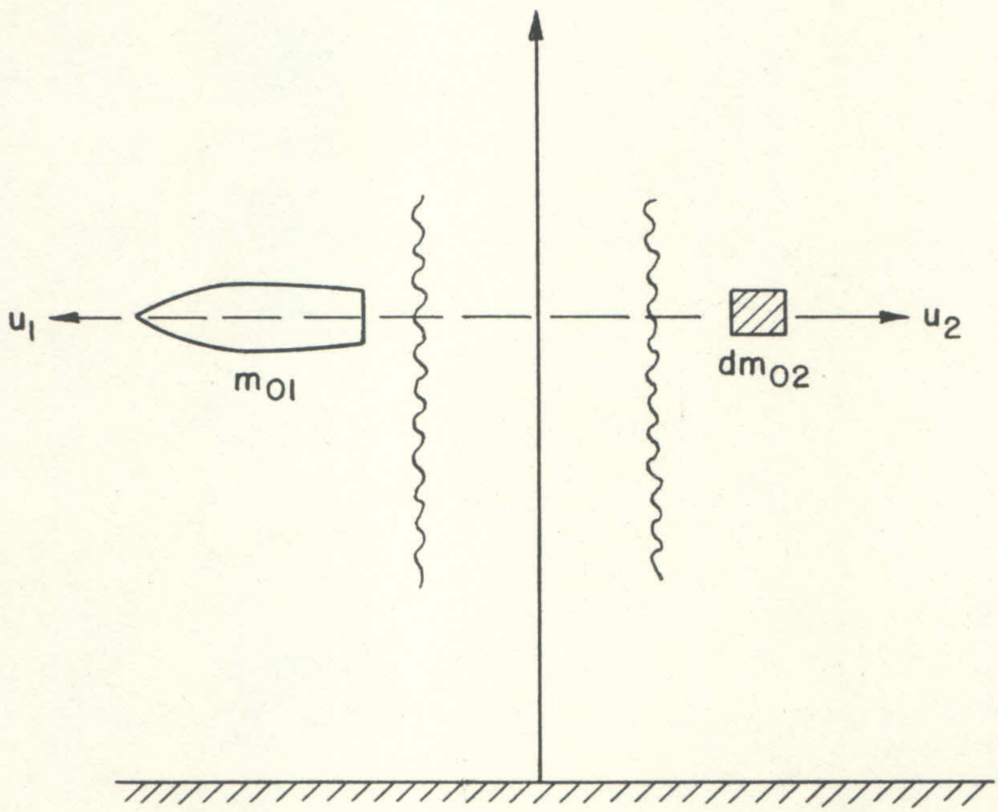


Fig. 55
H. S. Seifert
Rockets

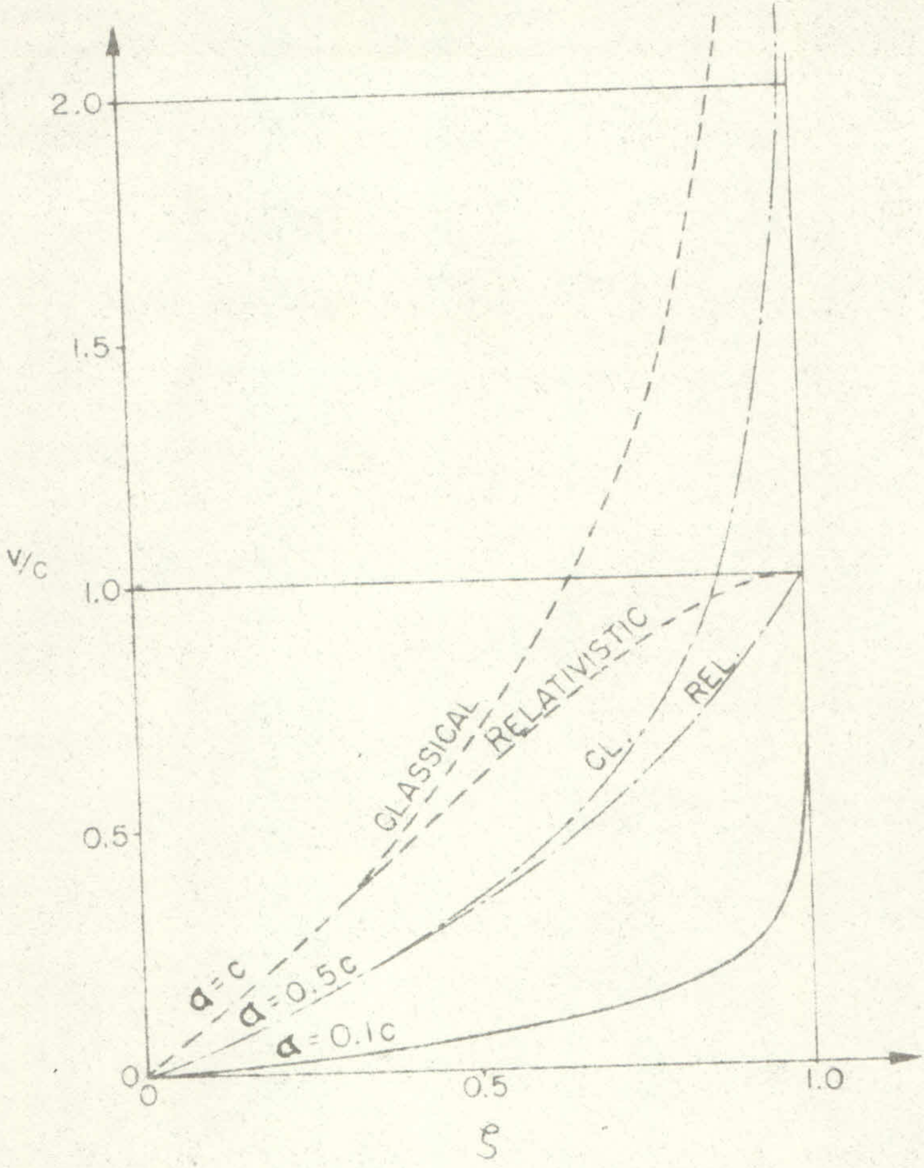


Fig. 56
H. Seifert
"Phys. of Rockets"